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THE INSTITUTION: ITS ORIGIN AND PROGRESS.

THE new Journal of The Institution of Civil Engineers marks another stage in the development of the publications of The Institution during its long life of over one hundred years and will, it is hoped, increase the facilities for "the acquirements of knowledge in engineering" which was laid down by the founders as one of the objects of The Institution.

For the benefit of those who receive for the first time a publication issued by this Institution, it may be of interest to recall the manner of The Institution's inception and to review its progress and its present scope of activities, and also the part played by Thomas Telford, its first President.

Early History.

It was on the 2nd January, 1818, that some of the abler and more enterprising of the younger engineers of the day met together at the Kendal Coffee-house, Fleet Street, opposite to St. Bride's Church, to discuss a Paper by Henry Robinson Palmer. An earlier meeting had been held at the same place on the 24th December, 1817, and it may be concluded that Palmer's Paper was the result of some agreement then arrived at to give concrete form to the discussion on the 2nd January. After the conclusion of this Paper, the meeting passed a number of resolutions moved by Palmer, the first of which was, "That a Society be formed consisting of persons studying the profession of a Civil Engineer."

The title "Civil Engineer" is often misunderstood by those who are not in close touch with engineering. The "Civil Engineer" has been aptly described as the "Engineer of Civilization" as it is he who opens up a country and provides and maintains the roads, railways, docks, waterworks, buildings, etc., which civilization demands. Perhaps in the beginning he was the "Citizen" Engineer and was so distinguished from his brother-worker the Military

Engineer. The term "Civil Engineering," as defined in the Institution's Charter, has, however, always embraced all branches of engineering, and so from the beginning The Institution has welcomed among its members all duly qualified engineers from all parts of the world. It is interesting to note that, whilst the founders were for the most part engaged in the construction of machinery, its membership to-day includes men famous in all branches of engineering as well as foreign engineers who have from time to time fulfilled the prescribed tests and have been admitted to membership.

At the next Meeting (on Tuesday, the 6th January), Field and Palmer were elected as Chairmen, and Jones as Secretary. A week later the Society formally adopted the title "The Institution of Civil Engineers," the first By-Laws were drawn up, and the practice of holding weekly meetings on Tuesdays, throughout a Session from November to March, was established.

Unfortunately the enthusiasm shown in starting this society was not maintained, and for a number of meetings the record shows "Not sufficient members present to form a meeting." It is not surprising, therefore, that on the 3rd February, 1820, the members resolved :—

> "That a respectful communication be made to Thomas Telford, Esq., F.R.S.E., Civil Engineer, requesting him to patronize The Institution by taking on himself the office of President of the same."

Telford accepted the invitation and this was the beginning of a long and fruitful existence for The Institution.

Constitutional Development.

Under Telford's fostering hand the young Institution grew steadily, though not without the natural troubles of settling its constitution, administration and finance upon satisfactory and permanent lines. In due course a decision was taken on the 15th January, 1828, to apply for a Royal Charter of Incorporation, which was granted on the 3rd June, 1828, and in connection with this it was necessary to define the profession of Civil Engineer. The Council of the society accordingly applied to Thomas Tredgold (an Honorary Member) to propose a suitable description. The result was the now well-known definition of Civil Engineering as "the art of directing the great sources of power in nature for the use and convenience of man," and this was embodied in the Charter. The first constitution provided for the election of Members and Honorary Members, and shortly afterwards a class of Corresponding Members ("beyond the limits of the threepenny post") was added. The first Corresponding Member, elected on the 23rd January, 1820,

was "Mr. Gibb, engineer, of Aberdeen," great-grandfather of one of the present Vice-Presidents of The Institution, whilst one of the earlier Members to join in 1821 was Bryan Donkin, under whom Palmer, one of the founders, served his apprenticeship, and here again a descendant, a great-great-grandson, is one of the Vice-Presidents to-day.

This constitution continued unchanged until December, 1837, when the Corresponding Members were merged into the class of ordinary Members; a differentiation was made in the subscription-rates according to whether a member was resident or non-resident, and a class of Graduates was established consisting of "persons pursuing a course of study or employment in order to qualify themselves for following the profession of a Civil Engineer."

In 1846 corporate privileges were withdrawn from Graduates and no further admissions took place into that class. Young engineers were then admitted as Associates, but the mixture of junior professional members with Associates who were not engineers by profession, was not found to be suitable, and in 1878 a class of corporate members, styled Associate Members, was established to distinguish them from Associates of whom those elected thereafter were not to have corporate rights. Previous to this, in 1867, provision was made for young men preparing themselves for the engineering profession to enjoy the advantages of association with The Institution by the establishment of a class of Students, enjoying many privileges but without corporate rights. The constitution thus arrived at has continued until the present time. Under the By-laws, however, based on the supplemental Charter in 1922, Members and Associate Members of The Institution are now styled "Chartered Civil Engineers."

In the early years three "Chairmen in Ordinary" of The Institution were elected, in addition to a Secretary and a Treasurer, Thomas Telford, as already stated, having been elected President in 1820. On the 21st March, 1825, the substitution of four Vice-Presidents for the Chairmen was decided upon and also the election of a Council of seven members, the President and Vice-Presidents being specially authorised to attend all meetings of the Council and to vote thereat. Further changes were made from time to time until at the present time there are elected annually by the members a President, four Vice-Presidents, and a maximum of thirty-one Members of Council, in addition to four Past-Presidents appointed annually to be Members of Council. Under the new By-laws which will come into operation shortly, a Member of Council elected after 1935, who has served continuously on the Council for five years, is not eligible for re-election, except as a Vice-President or President, until after an interval of at least one year. The great increase in the work falling upon the

Council made it impracticable for all questions to be fully and expeditiously dealt with at its Meetings, and so a number of Standing Committees of Members of Council have been formed. These Committees have been each allotted a certain branch of the administrative work of the Council and meet as often as is required to dispose of business. This ensures early and adequate consideration of all subjects, important decisions and recommendations being referred to the Council which retains complete responsibility for directing policy. The Standing Committees of the Council are the Finance, Publications and Library, General Purposes, Professional, Membership, and Education and Training. In addition, there are Institution Committees, such as the Local Associations and Research Committees, to deal on a continuous basis with the development and expansion of specific activities within the objects of the Charter, as well as Special Committees appointed by the Council from time to time to examine and report on specified matters.

Reference may be made here to the Benevolent Fund of The Institution of Civil Engineers. The charitable purposes of this Fund, however, form no part of the objects of The Institution, and accordingly its activities are carried out under the direction of an entirely distinct Committee of Management elected by and from members subscribing voluntarily to the Fund. The applications for assistance are dealt with on a system which ensures the strictest privacy, and, since 1864, approximately £254,000 has been spent in assisting about 900 cases.

Many world-famous men have held the position of President of The Institution, such as Robert Stephenson, Lord Armstrong, Sir Benjamin Baker, Sir John Wolfe Barry; whilst many other distinguished engineers, such as Isambard Kingdom Brunel, Sir William Siemens, Sir Joseph Whitworth, Sir Henry Bessemer, Sir William Thomson (Lord Kelvin), became Vice-Presidents or Members of Council, but for various causes did not reach the Chair.

On the 1st July, 1935, the Roll numbered 10,915, namely, Members 2,229, Honorary Members 16, Associate Members 6,930, Associates 55, and Students 1,685.

With the increase in the country membership of The Institution various Local Associations have been formed, where Papers are read and discussed and works of interest visited. The first to be constituted was the Glasgow Association of Students in 1884, followed shortly by the Manchester & District and the Birmingham & District Associations, and later by the Newcastle-on-Tyne & District, the Yorkshire, the Bristol & District, and the South Wales & Monmouthshire Associations. All of these, with the exception of the

Glasgow Association, have been reconstituted since the war as Associations of Corporate Members and Students, and in addition an Association was formed in Northern Ireland in 1933. Similar Associations have also been formed in Buenos Aires (1928), Malaya (1929), Shanghai (1932). It is hoped that this new Journal will form a link to keep these Associations in touch with The Institution and with each other.

Owing to the very wide distribution of members in all parts of the world, Advisory Committees have been formed in Canada, New South Wales, Victoria, South Australia, Western Australia, Queensland, New Zealand, South Africa, India and Malaya, to assist the Council in connection with the proposition-papers of intending candidates and other matters. The first of these Committees was initiated in 1890 in Victoria.

Overseas Members of Council have also been appointed to represent the interests of members who live outside the British Isles. These ordinarily number six, namely, two in India and one each in Canada, Australia, New Zealand, and the Union of South Africa.

The Institution Buildings.

The first Meetings of The Institution, as already stated, were held at the Kendal Coffee-house in Fleet Street, then for a brief period Gilham's Coffee-house, the situation of which is not known, was resorted to. In 1820, however, rooms were hired at 15 Buckingham Street, Adelphi. In 1834, a house (No. 1) in Cannon Row, Westminster, was taken, and four years later The Institution leased No. 25 Great George Street. This, with some alterations and additions made in 1846 and in 1868, sufficed for its needs until 1896, when it was rebuilt upon the sites of Nos. 24, 25 and 26 Great George Street. This building, however, was short-lived, as The Institution had to relinquish the site to the Government. Fortunately the circumstances were such as to simplify the task of securing a suitable site not too far away, and partly by exchange with the Government and partly by purchase, it secured the site on which the present building stands. Here, on the 25th October, 1910, the foundation-stone of the present building was laid by the late Sir James Charles Inglis, then President of The Institution. A reproduction of the artist's drawing of the present building is shown at the end of this note.

The Educational Policy.

The Institution has recognized, from its early days, its duty in connection with the education and training of those who apply to it for recognition, and the results can be seen in the high standard

which corporate membership is recognized as carrying with it to-day.

The first trace of action in this direction is found in the establishment in 1838 of a class of Graduates, although it was not until 1866 that the subject was brought strongly to the fore by Sir John Fowler in his Presidential Address, in which he dealt mainly with the means by which "the younger members and the rising generation" might best prepare themselves for the duties which the future would bring with it. The establishment of the Student class a year later again brought this matter into prominence, and in 1868 an inquiry was instituted by the Council into the methods, cost and effect of the systems of engineering education in use in different countries. A report was published in 1870 under the title "The Education and Status of Civil Engineers in the United Kingdom and in Foreign Countries."

In 1889 the Council turned their attention again to the qualifications of general education desirable for candidates seeking admission to Studentship. Thereafter they required evidence of a candidate's general education by means of certificates of certain examining bodies, and in 1891 they issued regulations governing the general education of candidates for Studentship.

By that time the examination system had become firmly established as a means of regulating the admission to other professions, and the question of requiring examination tests for election into The Institution arose. The initiative was taken by Sir John Wolfe Barry when he became President in 1896, and at his instance the Council caused an inquiry to be made as to the examination systems of other English professional bodies and of Continental engineering societies. The result of this inquiry was a decision to introduce examination tests for Studentship in subjects of general education and for Corporate Membership in the scientific subjects which are applied in engineering practice; the qualifications of a candidate in respect of practical training and experience, to which the Council have always paid special attention, continuing to be judged upon the testimony of the members supporting his application. It was also decided to hold a preliminary examination in suitable subjects of general education, for those who had not already passed a recognized test of that character.

In 1903 the Council appointed a Committee, under the Chairmanship of Sir William White, to consider the whole question of the education and training of engineers. The report, issued in 1906, confirmed in substance the suitability of the regulations of The Institution for those preparing for any branch of the Civil Engineering profession. In 1914 another Committee appointed by the Council,

under the Chairmanship of Dr. Unwin, reported on the Practical Training of Engineers, and it was as a result of this Report that the drafting of a form of joint undertaking on the part of the employer and pupil, in connection with the provision of suitable opportunities for practical training, was drawn up for use in cases where formal articles of pupilage or apprenticeship were not entered into. In 1919, the Associate Membership Examination, which then consisted of two Sections, A and B, was extended by the introduction of a test (forming Section C of the Examination) adapted to ensure proper acquaintance with the routine work of the engineer's office. This third Section was made compulsory for all candidates for Associate Membership; exemption from the other two Sections was allowed to holders of certain engineering degrees of the great majority of the Universities in the British Empire. A recent change now permits a candidate to submit a report or thesis as an alternative to the written test in "Engineering Drawings, Specifications and Quantities" which constitutes Section C.

Unremitting attention given to these Examinations in the endeavour to render them a satisfactory complement to the practical training and experience also insisted upon, has caused them to be generally accepted by employers, whether individuals, corporate bodies or Departments of State, and has resulted in the designation of corporate membership of The Institution of Civil Engineers being considered as the hall-mark of education and training in Engineering. One of the Standing Committees of the Council of The Institution continues to supervise closely this important work.

Allied with this work, and complementary to it, is the award of Medals, Premiums and Prizes for the best Papers submitted by corporate members during the Session, as this encourages members in their efforts to acquire knowledge and helps to disseminate it for the benefit of all. The most important of the Medals is the Telford Medal, which was first awarded in 1837. Similarly, suitable awards are offered for Students' Papers.

Professional Conduct.

Another duty undertaken by The Institution has been in connection with the mutual relations of the members as professional men and their relations with the public who employ them.

Even in the original rules of a hundred years ago, provision was made to remove any person whose action might be regarded by the Council as inimical to the interests of the general body, and whilst these provisions were amplified from time to time, it was not until 1910 that rules were formulated to govern the general conduct of the

members. The general purport of these rules is to ensure that the engineer's utmost skill shall be placed at the disposal of those who employ him, and that the method of his remuneration shall not involve any conflict between his personal interests and those of the clients whom he advises.

These rules have recently been further revised, after full and careful consideration by the Council, so that they may be suitable for present-day conditions.

Publications.

This Journal is the modern development of the steps that have been taken from shortly after the time Telford became President to disseminate knowledge necessary in civil engineering. It was in 1824 that Joseph Mitchell first took notes of the opinions expressed at the Meetings, and these formed the beginning of the published Proceedings of The Institution containing a repertory of almost all the great engineering works carried out since that time.

In 1834 a lengthy and elaborate code of rules and practice was drawn up, intended to ensure the compilation, under the direction of a "Committee of Discussions," of an accurate, concise and impartial digest of the discussions, with the supervision of an expert member specially appointed for each occasion.

In 1836 the first Volume of Transactions, containing selected Communications, was published by John Weale by subscription. A second similar Volume was published by Weale in 1838 and a third by The Institution itself in 1842. These three Volumes were in quarto size. Meanwhile, in 1837, the publication of "Minutes of Proceedings" in octavo form had been initiated, with the object of giving abstracts of Papers and Discussions, only certain selected Papers being reprinted in full in the quarto Transactions. The cost of the quarto volumes and other considerations led to a decision on the 12th March, 1844, to print all approved Papers in the "Minutes of Proceedings" as fully as possible.

The "Minutes of Proceedings" have remained unchanged in form down to the present time and constitute a series of 240 Volumes, of which the last two Volumes are in the press. In 1874 the publication in the Proceedings of other Selected Papers, *i.e.* those approved for publication but not deemed to be suitable for discussion, was undertaken. At the same time a third section was added comprising abstracts of Papers in foreign transactions and periodicals.

Soon after the war, in order to effect economies, the Council decided to publish Selected Engineering Papers and Engineering Abstracts separately and to devote the Minutes of Proceedings wholly to the

Papers read and discussed, together with the Presidential Address, the Annual Report and other proceedings of The Institution.

The Journal that now appears marks a break in former practice as regards the method of publication, but members can rest assured that the same standard to which they have been accustomed in the past will be maintained. It will, however, enable the early publication of Papers and the wide dissemination of the reports of the Research Committee, to which further reference is made on page 39.

Thus the Journal will contain the Papers read and discussed, which formerly appeared in the "Minutes of Proceedings," as well as "Selected Engineering Papers," which will not in future be published separately, and abstracts of other Papers contributed to The Institution which are available in the Library. It will publish the Reports from the Research Committee of The Institution and its Sub-Committees, in addition to Communications on Engineering Research received from other sources. It will also incorporate the Notices which were previously issued as "Sessional Notices," and record the principal work of Local Associations, so that all members will be able to keep in touch with the various activities of The Institution. No change will, however, be made for the present in the printing of Engineering Abstracts, which will continue to be issued separately.

A total of eight Numbers of the Journal, of which this is the first, will appear during the year, further Numbers being issued n the 15th December, January, February, March, April, June and October respectively.

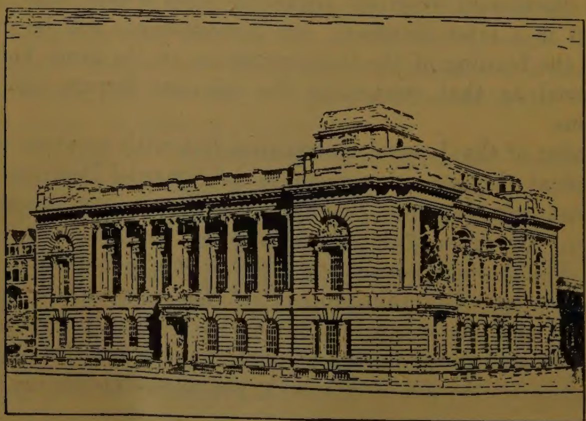
As a general rule, the Papers read at the Meetings in one month, with the Discussions upon them, will appear in the Number of the Journal for the succeeding month. The full Correspondence (*i.e.* written discussion) received regarding each Paper read will be published in a later Number; but arrangements will be made to allow of the binding of the Correspondence in the same Volume of the Journal as that containing the relevant Papers and verbal discussions.

The cover of the Journal maintains a link with previous publications, and at the same time includes the Armorial Bearings of The Institution. In heraldic language the latter are described as follows:—"Or on a Pale Azure between two Annulets in fesse Sable a Thunderbolt between in Chief a Sun in Splendour of the first and in base a Fountain proper." "The natural forces of heat, electricity, and water are symbolized respectively by the sun, thunderbolt, and fountain, the annulets representing the science of mathematics by the aid of which the engineer exercises his art and skill."

It is hoped that this short account of the inception and progress of The Institution of Civil Engineers will be of interest to the younger members, some of whom may not have had an opportunity of seeing the earlier reports issued.

Space has not allowed of the mention of many important contributions which The Institution has made for the advancement of engineering science, but a brief reference must be made to an undertaking initiated by The Institution which has been of great service to British industry, namely the work of what became the British Engineering Standards Association and is now the British Standards Institution. In January, 1901, on the motion of Sir John Wolfe Barry, the Council appointed a Committee to consider the advisability of standardizing various kinds of iron and steel sections. The result was the formation of a permanent Committee with the co-operation of the Institutions of Mechanical and Electrical Engineers and of Naval Architects and the Iron and Steel Institute, out of which has grown the present British Standards Institution, whose work has developed far beyond the scope of the original inquiry and which, since 1929, has had a Royal Charter.

Members can look back on a glorious past and also look forward with confidence to a future where opportunities will not be lacking for further contributions from Civil Engineers for the benefit of mankind. The Journal will endeavour to play its part in disseminating knowledge for the advancement of engineering science, and assist the Engineer in the many problems he has to face in his practical work by supplying him with the latest information on the work of engineering research bodies and with accounts of development in engineering practice.



JOHN DUNCAN WATSON.

ELECTED PRESIDENT 1935.

ORDINARY MEETING.

5 November, 1935.

Sir RICHARD AUGUSTINE STUDDERT REDMAYNE,
K.C.B., M.Sc.,
the retiring President, in the Chair.

Sir Richard Redmayne remarked that everything came to an end—even a Presidency of The Institution of Civil Engineers. By an excellent rule it was ordained that a President of The Institution should function for one year only, and he never returned. Before vacating the Presidential Chair in favour of a better man, he desired to say how grateful he was to the members for their consistent courtesy and kindness to him during his occupancy of that Chair. It was now his very great pleasure to introduce to the members their new President, Mr. John Duncan Watson. Mr. Watson and he had been elected to the Council in the same year, and had served on the Council together ever since. He knew full well that the prestige and best interests of The Institution were very near to Mr. Watson's heart, and he had not the least doubt that Mr. Watson would perform his duties as President with dignity and with great advantage to The Institution.

Mr. JOHN DUNCAN WATSON having taken the Chair,

Sir Charles Morgan, Past-President, moved the following resolution :—

> "That the members present at this meeting desire, on behalf of themselves and others, to record their high appreciation of the services rendered to The Institution by Sir Richard Redmayne during his term of office as President."

He only wished, he said, that the resolution might have been put in better hands than his own, because no words of his could form a really adequate appreciation of the benefit which The Institution had received from the retiring President. On all previous occasions when a President had retired, the members had heard his merits so far as they could be expounded at that moment; but Sir Charles had belonged to The Institution for very many years and he could honestly say that he knew of no President who had been so affable (if he might so express it) to all of the members not only of the Council but of the whole Institution, and who had done so much for the advantage and prestige of The Institution, as had Sir Richard Redmayne.

Sir Cyril Kirkpatrick, Past-President, in seconding the resolution, said that Sir Charles Morgan had put in kindly words the feelings of all the members. He himself did not think they could possibly have had a better President. Sir Richard Redmayne had carried out his duties without sparing himself; his had been a very hard

By adopting the new By-laws and by widening our outlook on the profession as a whole, The Institution has taken a step which may go far to encourage a more comprehensive outlook and create a better understanding of the interests pertaining to all engineers. A further change which I welcome is the introduction of a Journal which, while maintaining continuity with the previous publications of The Institution, will enable members to keep more closely in touch with the work that is being done to promote the science and practice of engineering. I take pride in the fact that the first Number appears during my year of office as President, but this decision of the Council to publish it was made during the presidency of Sir Richard Redmayne, to whom, and to Sir Clement Hindley, we all owe a debt of gratitude for the long-sightedness and for the energy they displayed in working out the problems involved in its first appearance.

It is our duty to help our sons to do greater and better things than their fathers, and therefore to encourage education in all its forms, to foster engineering literature, promote professional ethics, and establish good fellowship amongst all engineers. By establishing Local Associations in different parts of the country, The Institution took a good step forward fifteen years ago. The movement has progressed until there are now eight Associations in England, Scotland and Northern Ireland and three abroad—Buenos Aires, Malaya and Shanghai.

The first Local Association of Corporate Members and Students was formed in Birmingham, where much has been done to justify those of us who took part in its inauguration. Apart from the educational work which it originally set out to do, it encouraged friendliness amongst members and brought about conditions which are worthy of imitation elsewhere. No fewer than sixteen branches of engineering now meet at the James Watt Memorial Institute, and during the past session more than 200 lectures and committee-meetings were held on the premises.

In speaking to members of The Institution it is perhaps hardly necessary to stress the advantages to the profession of cultivating good fellowship; but when one has seen the benefits of it at home and experienced the happiness as well as the benefits of meeting foreign engineers on their own works it becomes a duty. To be invited by a foreign country to give professional advice is not only a compliment to the English engineer, but also a manifestation of good feeling and broad-mindedness which is worthy of emulation. I always look upon the invitations which I received from New York and Chicago as compliments which should not be forgotten.

I propose in this Address to follow to some extent the example

which so many of my predecessors have given and to speak about phases of Public Works which have engaged my attention for more than half a century.

I first became associated with public works as a pupil, and subsequently became assistant to James Watson, best known as Water-Works Engineer of Bradford. Later I became a Burgh Engineer for six years and a County Engineer for nine years. For more than twenty years I was engineer to a co-ordination of eight Local Authorities of which Birmingham was the predominant partner, and I have acted as consulting engineer to various Authorities in different parts of the world. Water-supply, sewerage and sewage-disposal problems have predominated, and it is to them that I now propose to direct attention.

Development of Sanitation.—At no time in the world's history has the necessity for a good water-supply been more universally recognized than it was in the later half of the nineteenth century. The ancient water-works of the cities of Greece and Rome were used chiefly to supply water to public baths, fountains and pools, and in a few instances to the palaces of the rich, but house-to-house supply as we know it to day, laid on to the dwellings of the poor and available in large volumes during every hour of the day and night, was undreamt of by the philosophers of Greece or the rulers of ancient Rome.

What we regard as the essential hygienic conditions of modern life were unknown even in the most celebrated cities during the Middle Ages, indeed it is less than a hundred years since the first complete sewerage system in the world was designed and carried out by one of our members, the elder Lindley, for the city of Hamburg in 1843.

Early last century English people suffered from cholera, typhoid fever and other infectious diseases to such an extent that those in authority became conscious of the fact that they were not free from responsibility for the manifestly unhygienic conditions which obtained in many parts of the country. The common people themselves were made to realize that the epidemics from which they had suffered were not acts of God, as they had too readily assumed them to be, but were the result of failure on their own part to give heed to what we now recognize to be the inevitable laws of Nature; and that freedom from disease accompanied the observance of these laws just as the converse brought suffering and death. The growth of their observance is reflected in the reduction of the death-rate during the last fifty years.[1]

Various important Acts were placed on the Statute Book from

[1] Appendix I (p. 34).

time to time, beginning with the Public Health Act, 1848. All served a good purpose, but the greatest stride forward was made possible by the construction of reservoirs, aqueducts, distribution mains and equipment in our great cities to supply water for the domestic and industrial needs of the inhabitants.

The Local Government Board, acting as supervisor of public health, and instigated by their Engineer, our Past-President, Sir Robert Rawlinson, encouraged all local authorities to follow the example of large towns and introduce good potable water into their respective districts. This lead was followed, and by the end of the 'sixties public opinion was almost unanimously in favour of spending money on water-supplies and sewerage. Florence Nightingale said that "the true key to sanitary progress in cities is water-supply and sewerage"; and Dr. Corfield, Professor of Hygiene and Public Health in London University, said, "We all know what water-carriage has done. We know that in towns where it has been introduced in conjunction with other sanitary improvements it has been the means of annihilating cholera," adding, "We know that it has been little less effectual in the extermination of typhoid fever."

When a cholera epidemic appeared in the early 'nineties the general public awakened to the necessity for more rigorous observance of hygienic laws. Hamburg and Altona adjoin one another on the right bank of the Elbe, and both towns drew their water-supply from that river. Altona, being nearer the sea, received river-water that had been polluted by the Hamburg sewage; yet cholera devastated Hamburg (there were 8,605 deaths in 1892) and Altona suffered but slightly (325 deaths). This was due to the Altona engineers having constructed sand filters which were able to eliminate pathogenic organisms from the river-water and to effect a general purification, whilst Hamburg did not filter the water at all. The publicity given to this case induced many local authorities in England to take action, when otherwise they would have held back. In this connection it may be noted that the revenue expenditure on sewerage, sewage-disposal, and waterworks by local authorities in England and Wales has increased from £4,359,295 in 1885 to £30,735,274 in 1933.[1]

One of my duties in Aberdeenshire in the 'nineties was to provide piped water-supplies to villages, followed in most cases by sewerage and in some cases works of sewage-disposal. An interesting and instructive phase of public health work arose out of a justifiable complaint which was made at this time by the citizens of Aberdeen to the effect that their water-supply, which was taken direct from the river Dee, was being polluted by sewage from several villages

[1] Appendix II (p. 34).

upstream. Before taking action the Town Council engaged two eminent bio-chemists, Professors Percy Frankland and Carnelly, to report upon the general effect of the alleged pollution. To the surprise of most people the professors reported that the water complained of was not less pure than the river at Balmoral 25 miles upstream. When it is remembered that the Royal Commission on River Pollution characterized the Dee at Balmoral as one of the finest potable waters in the world it is not surprising that the citizens of Aberdeen were astonished at their experts' report. Pettenkofer and Schenk in Germany and Professor Percy Frankland in England had all proved that a stream that had received sewage, and then flowed for miles over a rough bed absorbing dissolved oxygen as the result of being dashed from stone to stone on its way to the sea, formed an excellent illustration of the marvellous recuperative power of Nature. At that time even well-informed men were unaware of Pettenkofer's purification theory. The influence of this scientific discovery, combined with the Hamburg scare, induced the Town Council of Aberdeen to give up all idea of litigation. Instead they helped the villagers with money to provide sewage-farms to purify the sewage before it was discharged into the river.

At this time the lower parts of the Thames, the Clyde, and inland rivers like the Tame and the Calder were very repulsive to those who possessed sensitive olfactory nerves. One of the River Commissions reported that "with very few exceptions the streams of the West Riding of Yorkshire ran with a liquid more like ink than water," which reminds one of a letter addressed to a Royal Commission of this period. The writer of the letter said "that he was using river water to write with," adding "that he was sorry he could not send the odour with it."

Whilst far-seeing men like Sir Joseph Bazalgette, Sir Edwin Chadwick, Sir Robert Rawlinson, Baldwin Latham and Bailey Denton had extolled the use of water as a vehicle for carrying all sorts of putrescible and other objectionable matter away from a town, they deprecated its discharge into clean streams until it had been purified by percolation through suitable arable or pasture lands, and, where economically practicable, suggested direct discharge into the sea.

Purification by Dilution.—The Iddesleigh Commission, in their fifth and eighth reports, showed that the disposal of sewage into an adequate volume of clean water—both salt and fresh—was sound and proper not only on the grounds of efficiency and economy but on strictly scientific principles.

There are, of course, good ways and bad ways of giving effect to a sound proposition, and there has been an inclination sometimes

to forget that if Nature does not provide an efficient method of dispersing the sewage when it enters tidal water it may become necessary for the engineer to promote dispersion artificially. In other words, when it is known that oxidation of impure liquid depends upon the dissolved oxygen present in the diluent into which it is discharged, it would be foolish to discharge a great volume of sewage into comparatively still water without making provision for the most complete mixing and bacteriological facilities to convert organic matter into simple harmless inorganic compounds.

The American engineer has of necessity had to consider this aspect of the question more than we have in this country because he is so frequently obliged to discharge sewage-effluents into comparatively pure water. It is no uncommon thing to draw potable water from an American lake, into which a sewage-effluent is being discharged at some other point. In such a case quick and effective dispersion becomes essential, and it was to overcome one of the difficulties of such a case that the multiple outlet was designed.

One of the first of these multiple outlets was advocated in a joint report to the City of Toronto in 1909 by the late Dr. Hering and myself, in which we recommended the Local Authority to carry the partially-purified effluent into 30 feet of water by means of a pipe extending for a distance of 3,500 feet from the shore ; to facilitate oxidation this pipe was provided with small openings or branch-pipes. The Boston and Passaic Valley outfalls are very successful examples of this method, the latter having no fewer than 150 nozzles acting as diffusers. Where tides and currents exist near the outfall this quick mixing by multiple outlets may not be needed, but it certainly contributes in all circumstances to the fulfilment of a sound principle.

The City of Chicago has spent colossal sums to conserve the purity of its water-supply and to effect purification of sewage by dilution. Until thirty-five years ago the Chicago River laden with sewage debouched into Lake Michigan ; it was then diverted towards the west by cutting through the " divide " and making it flow towards the Illinois River. The main channel cost $24,000,000, and it was subsequently widened at a cost of $10,000,000 in order to make provision for abstracting an additional volume of clean water from Lake Michigan to dilute the sewage still more. The urgency for this was due to the abnormal increase of population and to the obvious failure of the dilution system, as it then existed. When this scheme was adopted our member Dr. Hering and his colleagues, Messrs. Williams and Artingstall, recommended dilution equivalent to 4 cubic feet of lake water per second to be discharged into the canal for each thousand of the population connected with the

sewers. For this to accord with the standard set by the Iddesleigh Commission in 1908 the volume of diluent should have been ten times as great.

Other Methods of Disposal.—Apart from dilution, Royal Commission after Royal Commission expressed the opinion that the only other way to purify sewage was to irrigate land with it, the famous Craigentinny meadows having shown the way to do so economically.

In 1871, Mr. Thomas Hawksley, our Past-President, recommended the Town Council of Birmingham to lay out a large sewage-farm in the valley of the Tame. His reports show that he was not unconscious of the limitations of irrigation when applied to growing towns, but at that time it was the only authentic method which an engineer was justified in recommending.

Parliament did not allow the Town Council to acquire the 1,000 acres of land for a sewage-farm which Mr. Hawksley regarded as necessary, and it was not until the present Drainage Board was formed and his son Charles, also a Past-President of The Institution, was authorized to prepare plans to support another Bill in 1897 that there was a prospect of the policy which was defeated in 1872 materializing.

The Birmingham, Tame and Rea Bill to acquire land became an Act of Parliament in 1897. This land was being under-drained for irrigation, when I was bold enough to suggest to Mr. Charles Hawksley the advisability of building some bacteria-beds of the percolation type on a scale that would establish or condemn irrevocably, as far as Birmingham was concerned, the alleged power of the bacteria-bed to purify sewage. So far from rebuking me, he encouraged me to recommend the Drainage Board to build an installation of bacteria-beds capable of treating a large part of the sewage for which his scheme had been approved by Parliament. This was a very magnanimous gesture on his part, considering that it was liable to misinterpretation.

By the later days of the century Messrs. Dibden, Cameron and others had made considerable progress with septic tanks and contact-beds; indeed the progress was so great that pressure was being brought to bear upon the Local Government Board by engineers and others to obtain reliable information to confute or to support their claims. The result was the appointment of the Royal Commission on Sewage Disposal in 1898 under the chairmanship of Lord Iddesleigh. The reason given for the appointment of the Commission was "that it is now contended that in many cases the land available is either of unsuitable quality, is available in quite inadequate area for effective filtration through the soil, or is obtainable only at prohibitive cost; and it is suggested

that sewage purification may in such cases be carried out on comparatively small areas artificially prepared."

This Commission sat for about seventeen years and issued nine most valuable reports which had the effect of placing sewage purification on a higher scientific plane than formerly. The first, dated 1902, stated, "We are satisfied that it is practicable, uniformly, to produce by artificial processes alone either from sewage or from certain mixtures of sewage and trade refuse . . . effluents which will not putrefy, which would be classed as good according to ordinary chemical standards and which might be discharged into a stream without fear of creating a nuisance." This finding was by no means unexpected, and it served as an *ex cathedra* statement upon which engineers might justifiably base recommendations to build bacteria-beds to take the place of land. Had it not been for the decided views of the Royal Commission it is very doubtful whether the towns represented by the Birmingham, Tame and Rea District Drainage Board would have agreed to such a complete change of policy, notwithstanding the fact that they were at that time respondents in an action for injunction raised with the consent of the Attorney-General by the Tamworth Town Council and others. The Court of Chancery granted this injunction; it was appealed against, and the Court of Appeal were unanimously in favour of quashing the injunction, notwithstanding the fact that there was no precedent for such a course. The Master of the Rolls stated, "The defendants in this case are a public body discharging a most important function, who have certainly not spared money. We were told that upwards of £500,000 had been spent on the work and they have done what they could—not only have they done what they could but actually instead of damnifying they have benefited the River Tame. Under these circumstances is it right to say that this body should go on discharging their duties with a sword over their heads in the shape of a perpetual injunction? I think not." This was upheld by the House of Lords.

A case like this, which lasted more than ten years, illustrates the great responsibility which local authorities and engineers assume when they recommend and execute works of this nature. This change-over was made, not because the Drainage Board regarded the principle of purification by land irrigation as bad, nor because they were dissatisfied with any part of the engineering works under construction, but because they felt it was their duty to see that future works were made as good as it was possible to make them in the light of modern discoveries. This striking change of policy, which was made in the early days of this century, attracted the attention of local authorities and engineers all over the world. It

induced an American professor to say that England had taken the lead in works of this kind. "If one asks the reason for this," he said, "a fitting reply is found in the old adage 'Necessity is the mother of invention.' The English rivers are small and the English cities are large and numerous."

The Sheffield experience fifteen years later was even more noteworthy in view of the information which had been made available by the Iddesleigh Commission in the interval. In December, 1905, the Local Government Board held an inquiry and finally approved of a contact-bed scheme for the treatment of Sheffield sewage. This work was completed and in full operation early in the year 1914, but it failed to give satisfaction; the question of finding a remedy or a substitute for contact-beds occupied the attention of the Town Council forthwith. In 1922, as the result of research work at Manchester by Messrs. Fowler, Arden and Lockett, and of successful tests on his own works by the late John Haworth, the Town Council, with the approval of the Ministry of Health (as the Local Government Board had become), resolved to incur the responsibility of substituting bio-aeration for contact-beds. One cannot speak too highly of the public service rendered by Sheffield and of the example they gave to other local authorities by resolving to abandon inefficient works and to substitute bio-aeration only eight years after their contact-beds were brought into use.

The attitude of the Local Government Board in approving this contact-bed scheme in 1905 was strictly correct, but it illustrates the need for what the Royal Commission had in mind when they suggested the initiation of a Central Authority, and it shows members of local authorities that they are not warranted in assuming that whatever is sanctioned by the Ministry of Health is necessarily the most expedient or the most economical. The Ministry make it clear that the responsibility for expenditure of public money rests primarily, if not solely, upon the local authority, whose business it is to engage an engineer in whom they have confidence to prepare a scheme which will give the best results for the least cost. It is not fully realized by the public that the Ministry's approval of a scheme does not imply that it is regarded as the best that could be designed or the cheapest that could be devised, nor that it embraces the most up-to-date methods available. The Ministry put forward the recommendations of the Royal Commission without saying that they must be acted upon. Therefore there is a danger of the general public regarding these recommendations—now more than a quarter of a century old—as requirements where sanction to borrow money is concerned. Further, the public often assume that when sanction to borrow money for a specific scheme is granted by the Ministry, it

becomes hall-marked (so to speak) and this assumption is accentuated when that sanction is given, subject to certain alterations to their engineers' plans.

Purification by Land-Irrigation.—As a matter of principle, purification by land-irrigation is sound and still fairly popular. When the soil is suitable and when the area is adequate, it is safe to say that the effluents from it in normal weather will be fit to enter a river. Irrigation or downward intermittent filtration may therefore be regarded as a reliable process; but care should be taken to see that the process is not out of harmony with its surroundings. For example, to retain a large farm near a town, when there are factories or dwelling-houses in the neighbourhood, would be as unwise as it would be uneconomical.

Contact-Beds.—Contact-beds followed and land-irrigation possessed the merit of concentrating an unpopular work on a small area. They still exist and continue to perform the work for which they were designed, but they lost favour with the public when the Royal Commission on Sewage Disposal pronounced them less economical than percolating filters.

Percolating Filters.—Percolating filters are distinctly popular. This method of purification has a direct relationship to the contact-bed, but it produces a more consistently uniform effluent, it is more generally reliable, and the operating costs are less. It is not, however, free from potential and sometimes actual nuisance. Where the filters occupy a large area it is advisable to locate them as far from dwelling-houses as possible, for one cannot forget that there are recurring periods of close, warm weather when fly nuisance and nauseating odours become so oppressive that residents in the locality complain. Notwithstanding all this, there is no gainsaying the fact that the percolating filter is popular with the engineer, the local authority, the manager of the works and last, but not least, the river authority.

Bio-Aeration or Activated Sludge.—The bio-aeration or activated sludge process provides the sanitary engineer with an important source of strength, when designing suitable works for special cases.

Although it was untried twenty years ago, it is now established beyond question to be one of the most useful methods of sewage-purification. It has proved itself to be scientifically sound, and when the plant is well designed, it is economical and freer from nuisance than any method yet discovered. It attempts to concentrate the process into the smallest possible space by bringing the sewage, the purifying medium, and the air into more intimate contact than is possible by any other known process of purification. At the same time it is too sensitive to be invariably reliable. In

some situations where inflowing sewage suddenly changes its character, as the result of a sudden inrush of troublesome trade waste, the efficiency is impaired. That it requires skilled attention to function successfully renders it unsuitable where works are necessarily small or isolated, but where conditions are suitable it supplies a want the value of which it is impossible to over-estimate.

Modern Development.—It has not been materially prejudicial to the solution of the sewage-purification problem that development has been comparatively slow—the sciences of biology, chemistry and engineering have worked harmoniously together, and the result has been sound and steady progress. My impression is that the next generation will do more to establish this science than the present generation has done. The knowledge that public money can be spent to yield a good return on works capable of promoting health and communal self-respect will do more to attain clean rivers than any compulsion that can be used.

The chief element which makes for progress in a democratic country is public opinion, and that usually concentrates upon what is desirable and what is readily attainable at reasonable cost. In terms of sewage-disposal, development on the following lines is indicated :—

(1) The adoption of more hygienic methods of sludge-disposal.
(2) The adoption of methods to obviate smell and fly nuisance.
(3) The adoption of more reasonable treatment of storm water.
(4) Co-ordination of Urban Districts in watersheds in order to minimize the number of sewage works and improve their character.
(5) Establishment of a central authority, or extension of the powers of the Ministry of Health to embrace the duties of the Central Authority proposed by the Royal Commission on Sewage Disposal, 1898.

Sludge Disposal.—In some districts the problem of sludge-treatment (referred to by Lord Bramwell's Commission as the crux of the question) is made exceptionally difficult owing to trade wastes; for example, in Bradford, where the predominating trades are associated with the woollen industries, the cost of sewage-treatment is 129·4*d.* per annum per head of population; in Burton-on-Trent, which specializes in the production of beer, the cost is 190·8*d.*; but in nineteen towns of England the average figure is 67·3*d.* It is not necessary to dwell unduly on this, but it is essential for the engineer to remember it when designing purification-works.

Early in the century I discovered by accident that digested sludge had no objectionable smell, whereas undigested sludge which had been buried ten feet deep for twenty years still retained its foul

smell. Evidence of the bare facts of this case was given to the Royal Commission in 1905. An attempt was made by me in 1902 to keep the sludge apart from the liquid sewage and to septicize the sludge by itself, but it failed. Dr. Travis and Dr. Karl Imhoff succeeded a few years later, and the latter designed and patented a two-storey tank, which became known as the Emscher tank, in which the upper storey induced sedimentation and the lower storey acted as a digestion tank.

For many years Birmingham was the only town to benefit from digestion of sludge, and I was under the impression that the principle was quite new when I tried and failed to put it in operation in 1902. Nearly thirty years later I learned about Walter East's British Patent Specification No. 92 of 1878 which says *inter alia*, "According to my invention I purify the sewage by retaining it in a covered tank, and whilst it is there I hasten the fermentation or putrefaction by adding to it a ferment containing suitable germs, for example, other sewage in a state of active fermentation . . . I provide a number of tanks each of a capacity to receive a day's sewage or somewhat more. . . . In starting the process I draw into the tank from one of the adjacent tanks a quantity of sewage already in a state of active fermentation. I then proceed to fill the tank adding to the fermenting sewage three or four times its bulk of fresh sewage."

When the Iddesleigh Commission issued their fifth report in 1908 they were not able to suggest a method of sludge disposal which had earned general approval. They suggested trenching into land; pressing for use as a fertilizer; pressing to facilitate burning; and dumping into the sea. With one exception they were all productive of complaints, and even that one—dumping at sea—was not favoured by several eminent men; for example, when Sir William Crookes delivered his presidential address to the British Association in 1898, he said, "We cannot afford to throw away products containing fixed nitrogen to the value of no less than £16,000,000 per annum." Liebig went further and said, "Nothing will more certainly consummate the ruin of England than a scarcity of fertilizers —it means a scarcity of food. . . . It is impossible that such a sinful violation of the Divine Laws of Nature should for ever remain unpunished, and the time will probably come sooner for England than for any other country when with all her wealth in gold, iron and coal she will be unable to buy one thousandth part of the plant food which she has during hundreds of years thrown recklessly away."

The War unfortunately brought the work of the Iddesleigh Commission to an end before it had the opportunity of investigating the question of sludge digestion. After the War the Government

appointed a Committee to investigate recovery of sludge products from the economic standpoint, and that Committee expressed the definite opinion that ordinary tank sludge as it is usually given or sold to a farmer does possess sufficient nitrogen to give it a definite fertilizing value; they also suggested that nitrogen may be imported for less than the monetary value of sludge delivered at the farm; in other words, the sewage-works must adjoin the farm if it is to pay.

The chief merits of the sludge-digestion process may be stated briefly as follows:—

(1) Removal of malodour effectively, and elimination of any tendency for the sludge to revert to its original condition.
(2) Reduction to an end product which is readily dried.
(3) Reduction by 50 per cent. (by volume) of sludge as the result of fermentation and gasification.
(4) Production of gas (chiefly methane) for power or light.

Before digestion was adopted in certain cases, it was found that the very liquid sludge, containing only 2 per cent. or less of dry solid matter, which emanates from the bio-aeration process was so bulky and troublesome to get rid of that it sometimes acted as a deterrent when a change from bacteria-bed treatment to bio-aeration was being considered. When, however, the digestion process was developed it was found that this liquid, or surplus activated-sludge liquor, could be mixed with crude sludge to the advantage of both.

It has not been wholly prejudicial to the solution of the sludge purification problem that development has been comparatively slow, but at the same time in looking back it is somewhat humiliating to realize that sufficient methane gas had been drawn off a septic tank at Exeter in 1895 to illuminate the sewage works there, and yet we did nothing to utilize this valuable power-gas produced from the sludge-digestion process.

Although this fermentation or digestion process had been operating at Birmingham for some years, the practicability of utilizing the available gas which was obviously being wasted did not occur to me until I visited a leper asylum near Bombay in 1919 where there were the remains of a small plant (1½ H.P.) which had been designed to operate from a septic tank by Mr. C. Carkeet James. On returning to England I obtained an opportunity to build a gas-collecting installation at Birmingham, which served a 35 HP. engine. This, in turn, led Mr. H. C. Whitehead at a later date to build at the Birmingham main works the largest installation of its kind in the world, a description of which was given to The Institution in a Paper by Messrs. F. C. Vokes and C. B. Townend in 1928, the Paper gaining a Telford premium. If the whole of the sludge from Birmingham

sewage were utilized for the production of power-gas it would yield 12 million HP.-hours per annum as against 4½ million HP.-hours, which is the present output.

In dealing with a subject of this kind it should be understood that a local authority's first duty is to dispose of noxious matter without creating a nuisance. The digestion method succeeds in doing this, but it should not be assumed too readily that it is the only way of effecting that object. The possibilities of betterment should form a permanent feature in the engineer's outlook.

Methods of Obviating Smell and Fly Nuisance.—One of the benefits accruing from bio-aeration which was not contemplated when the activated-sludge method of purifying sewage attracted attention (between 1915 and 1920) was its ability to free sewage from smell.

When I was asked in 1912 whether Ward's Island would be an acceptable site for an installation of bacteria-beds to purify the sewage of New York I was obliged to say "No." Recollections of smell nuisance due to spraying septic sewage over bacteria-beds in England during the hot summer of 1911 and the realization that no one knew how to control it compelled that negative answer.

A method of obviating malodour was discovered as the result of knowledge and experience emanating from the bio-aeration process. This was taken advantage of at Birmingham by installing what was called a bio-flocculation tank, in which the sewage-effluent from ordinary sedimentation-tanks was subjected to treatment for about one-sixth of the time required for full treatment of ordinary weak sewage, and the resulting liquor was found to be entirely free from colloids and apparently from smell.

After twelve years' experience it is safe to say that the bio-flocculation process hardly ever fails to remove all smell from sewage; incidentally, this can be done practically without additional cost where bacteria-beds are employed to effect nitrification, owing to the fact that a bacteria-bed is capable of purifying double its normal volume of liquor when the sewage is devoid of colloidal matter.

As I have referred to Ward's Island, New York, it is encouraging to record that the authorities, having been satisfied with the proof put forward that purification can now be effected without creating smell nuisance, are busy constructing on the island what may prove to be the largest activated-sludge plant in the world.

When a local authority is obliged to have sewage-works in proximity to dwelling-houses this question of smell is likely to produce more complaints than pollution of rivers, because the ratepayer is ever present, whereas the River Inspector is only an occasional visitor! It is expected that the reliable evidence of the efficiency of up-to-date methods of purification will make it easier to find suitable sites

for large sewage works, but sentiment is still very powerful and when objections of this nature are put forward with all sincerity they have great influence. This fear was illustrated when the Middlesex Act, 1931, was before the House of Lords. The Yiewsley and West Drayton Urban District Council became alarmed when they learned that the County Council intended to treat and dry, on farm land in their district, all the sludge of West Middlesex. They were persuaded to visit the Birmingham Works and see for themselves how the digestion process operated before they incurred the expense of opposing the Bill. The day selected for their visit turned out to be the hottest day of the year, and when they saw the works and visited the drying area, which showed sludge in all stages of treatment over an area of 80 acres, they were convinced that the statements which had been put forward by the promoters of the Bill were not exaggerated. It should be recorded also that they were fair-minded enough to admit this fact at their next public meeting.

Now that both sewage and sludge can be economically deodorized on a great scale without giving rise to objectionable smell, it may be assumed that we have reached a stage when the general public will demand from local authorities works capable of giving a higher standard of service in this respect than hitherto.

The fact that the digestion and bio-aeration processes are accepted as hygienically sound and are generally practicable is not *prima facie* evidence that we have reached the goal, but it is a very great stride forward and one which an engineer can recommend a local authority to adopt.

Treatment of Storm-Water.—That a local authority should incur the expense of purifying the whole flow of sewage in dry weather is accepted by everyone, and most people admit that three times that volume should be treated in like manner in time of rain, but how much more ought to be treated when rainfall is excessive is open to question. The volume is not laid down by statute, but the object of those who framed the law is quite apparent to all, and the recommendations made by the Royal Commission in 1908 have proved to be generally acceptable to Rivers Authorities and Fishery Boards; they have also had their influence upon Courts of Law. It is, however, even more of importance to the engineer when he is called upon to design a scheme that he should know what volume the Ministry of Health regard as essential. The Ministry have accepted the Royal Commission's recommendations that local authorities should provide works capable of purifying three times the normal dry-weather flow and that they should partially purify an additional three times the dry-weather flow before the surplus is allowed to flow

uncontrolled to the river. If these recommendations were given effect to in the manner contemplated by the Commission our rivers would be much cleaner than they are.

Having been a Conservator of the River Trent, and having as an engineer been largely responsible for expenditure of public money in order to effect purification of our rivers, I am anxious that the value of that work should not be thrown away or impaired by allowing what is mis-named storm-water to foul our rivers until six times the dry-weather flow is dealt with. It is too readily assumed that storm-water, because it is chiefly rain-water, is always weaker than the ordinary dry-weather flow, but this is not so. To use the words of the Royal Commission, "the effect of a storm is to increase for the first hour or more the amount of organic matter in solution in the sewage as it arrives at the works, and also very largely to increase the suspended solids."

Greater use should be made of automatic recorders to gauge (1) all the sewage arriving at sewage works, (2) the volume undergoing full treatment up to three times the dry-weather flow (as recommended by the Royal Commission), (3) the volume undergoing partial treatment by the mechanical process of settlement only.

Too little attention is given to the Ministry of Health's declaration that "in the absence of any special circumstance overflow weirs should be fixed so as not to come into operation until the flow exceeds six times the dry-weather flow." County Councils, who are the Rivers Authorities in the great majority of cases, rarely trouble themselves about it, and the Ministry take no steps to see that their views are enforced.

The least costly item in building sewage-purification works is the construction of impounding tanks; yet there is a tendency to make storm-water tanks too small. The Royal Commission suggested that storm-water tanks capable of holding six hours' normal dry-weather flow might be adequate, but this suggestion was made in 1908 when waterbound macadamized roads prevailed. To-day roads and back-yards are more or less impervious, so that surface-water not only reaches a sewer in less time than formerly, but it contains more grease and organic matter than it did. Although the Royal Commission did not advocate the high standard set by the Public Health Act, 1875, their recommendations are regarded as reasonable. To-day, no authority is justified in curtailing the size or limiting the efficiency of a storm-water impounding tank merely because the Commission suggested when circumstances were different that its size might be limited to a six-hour capacity.

Lack of Co-ordination.—Some years ago, when speaking at Manchester, the Earl of Derby referred to the lack of co-ordination

amongst local authorities and quoted remarkably impressive figures. He said that within a 15-mile radius of Manchester Town Hall there were no fewer than fifty-one local authorities or companies supplying gas, thirty-five supplying water, thirty supplying electricity, twenty-nine providing tram and omnibus services, and no fewer than one hundred controlling sewerage and sewage-disposal. Such figures have had great influence upon the minds of business men who are members of local authorities. They have emphasized the need for action, and justified the Government in establishing the electric grid on a sound basis all over the country. Lord Derby did not suggest that his proposals were novel, but he did stress the need for long-sightedness in dealing with public works.

A co-ordination to deal with sewerage and sewage-disposal was formed at Birmingham when Mr. Joseph Chamberlain was Mayor of that city in 1875. It has been a great success, and, although the Birmingham city boundaries were extended in 1909 to embrace one borough and four urban districts, the Birmingham, Tame and Rea District Drainage Board continues to be a local authority representing one city, one county borough, one Royal borough, one urban and one rural district. Chamberlain's lead was followed by others both in England and America, the most recent development being in Middlesex.

In supporting the third reading of the Middlesex Act, 1931, the Minister of Health said, " This is a £5,250,000 scheme which has resulted from proposals made by the Government to local authorities last year." In other words, the Minister looked upon the Middlesex Act as the outcome of a suggestion that local authorities might under the Local Government Act, 1930, call a County Council to their aid rather than set up a new *ad hoc* body like that at Birmingham. This suggestion is helpful, but it should go further than call upon one county, because a watershed suitable for a joint scheme may embrace several counties. It was so in the case of the Birmingham Board, where several districts were in Warwickshire, two in Staffordshire and one in Worcestershire.

Of even more importance in this respect is the recent Report to the Minister of Health on Greater London Drainage, which was issued to the public in March last. It was signed by three members of The Institution, namely Mr. J. R. Taylor (representing the Ministry of Health), and Sir George Humphreys, Past-President, and Mr. Peirson Frank, representing the London County Council. It sets forth proposals of a far-reaching character, which are more or less in accord with the principles upon which the Middlesex Act, 1931, was framed.

The engineers who signed the Report summed up their recommendations in the following terms :—

(i) That all arrangements for sewage-disposal in and around the Metropolis should be co-ordinated and planned with regard to an area of 25 miles radius (or thereabouts) from Charing Cross.

(ii) That for this purpose consideration should be given to a scheme whereby the whole of this area would be served by ten, or fewer, centralized disposal works.

(iii) That, within this area, all further main sewerage and sewage-disposal works, or extensions of existing works, should be designed so as to conform with the foregoing.

(iv) That special measures should be taken to give effect to the above recommendations.

Of the ten purification works suggested it may be said that three are already in being: one at Heston and Isleworth (Middlesex), one at Barking, and one at Crossness. Others are receiving consideration from members of County and Town Councils adjacent to the Metropolis, who appreciate the wisdom of the suggestions contained in the Report and the need for helping small towns that are too ready to encourage the erection of works or factories within their areas without realizing that resulting trade-waste unmixed with a sufficient volume of domestic sewage is very difficult and expensive to purify.

Rivers Boards, such as that of the West Riding of Yorkshire, have also emphasized the need for co-ordination, and their decided convictions are influencing public opinion.

Establishment of a Central Authority.—The nation owes a great deal to the Ministry of Health and to their engineers for the way in which they have guided local authorities in times of trouble and strengthened them in their efforts to obviate pollution of rivers, but there is still much to do before the goal is reached. The cure for river-pollution in an industrial country like ours cannot be effected by a single prescription. Trade-waste is often a dominating factor, and it is a commonplace to say that there are no two sewages alike; but it is equally true to say it of rivers, and it is impossible to frame a standard of purity which would apply to all cases. It would obviously be a waste of public money to insist upon the same standard of effluent from a city like Liverpool, situated upon the estuary of a great river, as from a city like Birmingham situated on a river so small that the volume of the effluent is far in excess of the normal flow in the river itself.

The Royal Commission recommended the establishment of a Central Authority to deal with both normal and abnormal cases all over the country. This suggestion has not so far been acted upon.

That some authority other than the Courts of Law should have power to specify standards of purity in specific cases is admitted, but whether that authority should be the *ad hoc* body suggested by the Royal Commission or whether it should be the Ministry of Health itself is a matter of opinion. Personally, I think it should be the Ministry, provided it is given sufficient power and funds to set up a Research Department wide enough to include the excellent work which has been done in recent years under the ægis of the Water Pollution Research Board. The duties and scope of such a Central Authority will require great consideration, but its establishment would help local authorities and engineers all over the country to bring about a better state of rivers and water-courses than we have had since the water-carriage system came into being.

Sir HENRY MAYBURY said that those present would agree that they had had a wonderful Presidential Address, and would be glad to show their appreciation. Mr. Watson had begun by referring to himself as one of the "old brigade," who had been a long time in harness; there seemed to be no deterioration up to the present, and all would hope there would not be for many years. He had pleasure in moving the following resolution:—

> "That the best thanks of The Institution be accorded to the President for his Address, and that he be asked to permit it to be printed in the Journal of The Institution."

Mr. S. C. LEWIS, in seconding the resolution, said it was a peculiar pleasure to him to see Mr. Watson occupying the Presidential Chair, because over a long period of years during which he had been engaged upon works for supplying the City of Birmingham with water from the hills of Wales, Mr. Watson had been engaged in constructing works and perfecting processes for dealing with that water after it had been used, and the success he had met with in those works had won him deservedly a world-wide reputation.

The resolution was carried by acclamation.

The PRESIDENT thanked the members very warmly for their vote of thanks, and for the way in which they had received his Address. He was conscious that he had not dealt with a great many important phases of his subject. For instance, he had only just referred to the great work which was going on in Middlesex, as a Presidential Address was not subject to discussion, and he felt sure that members would wish to express a great many different opinions. It was hoped, however, that during the Session his son would present a Paper on that subject, which would give an opportunity for discussing the various aspects of the work.

APPENDIX I.

General Death Rate per Thousand.

	1884	1934		1884	1934
Birmingham	21·4	11·2	Manchester	26·4	12·6
Bristol	18·4	10·9	Newcastle	23·1	12·8
Cardiff	24·4	12·4	Norwich	21·2	11·5
Exeter	18·4	11·6	Plymouth	21·0	12·5
Hull	21·1	11·4	Sheffield	22·4	11·3
Liverpool	25·2	13·2	Southampton	17·1	12·0
London	20·3	12·2	Swansea	18·9	11·7
Leeds	24·2	13·0	York	21·1	11·6

Note.—In 1884 districts and sub-districts were taken as approximately representing the towns.

APPENDIX II.

Local Authorities in England and Wales. Sewerage and Sewage-Disposal Expenditure,

Year ending 31st March.	Capital Expenditure.	Expenditure on revenue account.	Outstanding loan debt at end of year.
	£	£	£
1885	985,385	1,866,671	16,569,353
1895	1,756,946	2,634,238	23,734,738
1905	2,895,162	4,230,691	37,608,900
1915	1,832,889	5,756,045	44,266,853
1925	5,039,587	8,710,545	49,241,397
1933	8,379,361	11,089,606	74,829,638

Waterworks Expenditure

(Including Metropolitan Water Board in 1905 and later years).

Year ending 31st March.	Capital Expenditure.	Expenditure on revenue account.	Outstanding loan debt at end of year.
	£	£	£
1885	1,231,598	2,492,624	30,326,906
1895	1,818,687	3,512,589	43,970,490
1905	†37,559,421	7,355,109	†114,698,642
1915	1,883,762	9,410,958	131,839,726
1925	6,665,815	15,839,483	148,059,326
1933	4,333,546	19,645,668	168,555,216

† The year 1905 includes in respect of the Metropolitan Water Board (operated from 1904) :—

	£
Capital expenditure	34,117,945
Loan charges	874,271
Other expenditure	3,355,396
Outstanding debt	34,943,557

MEDALS AND PREMIUMS.

The PRESIDENT presented on the 5th November a Telford Gold Medal, and the Awards for Session 1934–1935 were announced as follows :—

For Papers Read and Discussed at the Ordinary Meetings.

Frederick William Furkert, C.M.G., being ineligible as a Member of Council to receive an award for his Paper on "Remedial Measures on the Arapuni Hydro-Electric Scheme of Power Development on the Waikato River, New Zealand," the Council have expressed to him the thanks of the Institution.

1. A Telford Gold Medal to Bo Manne Hellstrom, M. Inst. C.E., for his Paper on "The Perak River Hydro-Electric Power Scheme."
2. A Telford Premium to Frederick William Daniel Davis, M.Sc., M. Inst. C.E., and William MacKenzie, B.Sc., M. Inst. C.E., jointly, for their Paper on "Major Improvement Works of the Port of London Authority, 1925–1930."
3. A Telford Premium to Geoffrey Lancaster Groves,[1] B.Sc., M. Inst. C.E., for his Paper on "Lambeth Bridge."
4. A Telford Premium to Charles Seager Berry,[2] M. Inst. C.E., and Arthur Creswell Dean, M.C., M.Sc., M. Inst. C.E., jointly, for their Paper on "The Constructional Works of the Battersea Power Station of the London Power Co., Ltd."
5. A Telford Premium to Ernest James Buckton, B.Sc. (Eng.), M. Inst. C.E., for his Paper on "The Construction of Haifa Harbour."
6. A Trevithick Premium to George McIldowie, Assoc. M. Inst. C.E., for his Paper on "The Construction of the Silent Valley Reservoir, Belfast Water-Supply."
7. A Manby Premium to Maurice Augustus Ravenor, M. Inst. C.E., for his Paper on "The Laws of a Mass of Clay under Pressure."
8. A Trevithick Premium to William Thomas Ward Miller, M. Inst. C.E., and Reginald Josiah Sarjant, M.Sc., jointly, for their Paper on "The Evolution of Various Types of Crushers for Stone and Ore, and the Characteristics of Rocks as affecting Abrasion in Crushing Machinery."

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[1] Has previously received a Telford Premium.
[2] Has previously received a Telford Premium.

9. A Crampton Prize to Vernon Ferdinand Bartlett, B.Sc. (Eng.), M. Inst. C.E., and William Henry Cadwell, Assoc. M. Inst. C.E., jointly, for their Paper on "The Coaling Jetty, Circulating Water System and Cable-Tunnels at the Battersea Power Station of the London Power Co., Ltd."

10. A Telford Premium to William James Horsburgh Rennie, M. Inst. C.E., for his Paper on "The Construction of the Chenderoh Water-Power Plant of the Perak River Hydro-Electric Power Scheme."

11. A Telford Premium to Arthur William Henry Dean, M.C., B.Sc. (Eng.), Assoc. M. Inst. C.E., for his Paper on "The Construction of a Submergible Road-Bridge over the Nerbudda River, near Jubbulpore, Central Provinces."

12. A Telford Premium and an Indian Premium to Matthew George Platts, O.B.E., M.C., B.Sc., M. Inst. C.E., for his Paper on "The Pykara Hydro-Electric Development."

13. A Telford Premium to John Alexander King Hamilton, B.Sc., Assoc. M. Inst. C.E., and John Tudor Graves, jointly, for their Paper on "Tees (Newport) Bridge, Middlesbrough."

14. A Telford Premium to Alexander Gray, M. Inst. C.E., for his Paper on "Saint John Harbour, New Brunswick."

For Selected Engineering Papers.

(*Being Original Communications ordered by the Council to be published without Discussion.*)

1. A Telford Premium to Thomas Hollis Hopkins, O.B.E., M. Inst. C.E., for his Paper on "The Bridges of the Egyptian State Railways."

2. A Telford Premium to Professor Ernest George Coker,[1] M.A., D.Sc., F.R.S., M. Inst. C.E., and Gilbert Peddie Coleman, B.Sc., Assoc. M. Inst. C.E., jointly, for their Paper on "Photo-Elastic Investigations of Shear-Tests of Timber."

3. A Telford Premium to Ralph Poole, Assoc. M. Inst. C.E., for his Paper on "The Theory and Design of Propeller-Type Fans."

4. A Telford Premium to William Harley Weston, B.Sc., Assoc. M. Inst. C.E., for his Paper on "The Construction of the New Sea-Locks of the Crinan Canal."

5. A Telford Premium to Charles George Watson for his Paper on "Catenarian Functions."

[1] Has previously received a Telford Gold Medal, a Telford Premium and a Howard Quinquennial Prize.

For Papers Read at Students' Meetings in London and by Students before Meetings of Local Associations.

1. The James Forrest Medal and a Miller Prize to James Halliday, B.Sc., Stud. Inst. C.E., for his Paper on "The Construction of a Reinforced-Concrete Grain Warehouse at Leith Docks."

2. A Miller Prize to Ronald Stephenson Cogdon, Stud. Inst. C.E., for his Paper on "The Reconstruction of the Quay Wall of No. 11 Berth, Hendon Dock, Sunderland."

3. A Miller Prize to Geoffrey Wood, B.Sc. (Eng.), Stud. Inst. C.E., for his Paper on "The Design and Construction of the Reinforced-Concrete Gantries and Retaining-Wall at Messrs. Stewarts and Lloyds' New Steelworks, Corby."

4. A Miller Prize to Reginald George Rowbotham, Stud. Inst. C.E., for his Paper on "The Application of Graphs to Reinforced Concrete and the Simplifying of Reinforced-Concrete Design."

5. A Miller Prize to Gordon Robert Coles, Stud. Inst. C.E., for his Paper on "A New Type of Gravity Dam Constructed for the Krängede Hydro-Electric Power Scheme."

6. A Miller Prize to James Louis Matheson, M.Sc., Stud. Inst. C.E., for his Paper on "Aerial Ropeways."

Bayliss Prizes.

Bayliss Prizes, awarded on the results of the October, 1934, and April, 1935, Examinations, respectively, to Hugh Mackintosh Irving, B.Sc., Stud. Inst. C.E., and James William Milne, Stud. Inst. C.E.

Charles Hawksley Prize.

The Council have awarded a Charles Hawksley Prize of £150 to Frank William Curry, Assoc. M. Inst. C.E., for his design of a railway station.

Coopers Hill War Memorial Prize.

The Council have awarded the Coopers Hill Prize for Session 1934–1935 to Arthur William Henry Dean, M.C., B.Sc. (Eng.), Assoc. M. Inst. C.E., for his Paper on "The Construction of a Submergible Road-Bridge over the Nerbudda River, near Jubbulpore, Central Provinces."

ADDRESS TO H.M. THE KING.

Fifteen Engineering Institutes joined by invitation with The Institution in sending to His Majesty the King the following Loyal Address of Congratulation on the occasion of the Silver Jubilee :—

TO THE KING'S MOST EXCELLENT MAJESTY.

May it please Your Majesty.

We, as representatives of, and on behalf of the principal Engineering Institutions and Societies in the United Kingdom of Great Britain and Northern Ireland, humbly beg leave to convey to Your Majesty the Loyal Greetings and Felicitations of our 67,000 Members upon the attainment of the 25th Anniversary of Your Majesty's Accession to the Throne.

Through Your Majesty's wise and beneficent rule and encouragement of Science in Industry, the utilization of the great sources of power in Nature has been advanced to an extent unprecedented in the history of mankind, thus contributing in no small degree to the welfare of all Your Majesty's subjects.

We desire to express our deep loyalty and devotion and our high appreciation of the work Your Majesty has done towards the promotion of the Science and Art of Engineering in its bearing on the well-being of the Empire.

We pray that Your Majesty may live long to enjoy good health and happiness.

THE INSTITUTION RESEARCH COMMITTEE.

As already indicated in the Annual Report of The Institution (1934–35) the Council have reconstituted the Research Committee of The Institution.

While The Institution has in the past been responsible for the initiation of many valuable researches, the Council have been impressed with the growing volume of problems where the engineer in his practical work lacks the assistance which science can give him. Progress in technical methods and in design is too often hampered by ignorance of fundamental scientific facts and the Council are convinced that The Institution should play a more active part in enabling these gaps in scientific knowledge to be filled by research. As a first step in undertaking this more active rôle the Research Committee has been constituted on a wider basis, including engineers drawn from the whole membership of The Institution. The members of the Committee are:—

Sir Clement D. M. Hindley, K.C.I.E., M.A. (*Chairman*).
David Anderson, B.Sc.
W. J. E. Binnie, M.A.
Professor Gilbert Cook, D.Sc.
Professor W. E. Dalby, M.A., B.Sc., F.R.S.
Sir Eustace H. Tennyson d'Eyncourt, Bart., K.C.B., LL.D., F.R.S.
S. B. Donkin.
H. L. Guy.
Sir Robert A. Hadfield, Bart., D.Sc., D.Met., F.R.S.
W. T. Halcrow.
R. G. Hetherington, C.B., O.B.E., M.A.
Professor C. E. Inglis, O.B.E., M.A., LL.D., F.R.S.
Professor A. H. Jameson, M.Sc.
G. G. Lynde.
David Lyell, C.M.G., C.B.E., D.S.O.
Professor K. Neville Moss, O.B.E., M.Sc.
A. Newlands, C.B.E.
Professor A. J. S. Pippard, M.B.E., D.Sc.
Professor Andrew Robertson, D.Sc.
Sir Leopold H. Savile, K.C.B.
R. H. H. Stanger.
R. E. Stradling, C.B., M.C., Ph.D., D.Sc.

The terms of reference to the Committee include a general mandate authorizing a survey of the whole field of engineering research and make the Committee responsible for recommending to the Council suitable subjects for research and for the direction and control of such work as may be undertaken. These terms were made as wide as possible so that the Committee might have the requisite freedom

in selecting subjects for research while avoiding overlapping or duplication of the work being done by other scientific and specialized bodies. It is the express wish of the Council that the Committee should seek co-operation with other Institutions and bodies connected with Engineering wherever possible and that whatever research is considered necessary it should normally be entrusted to such existing research organizations as may be equipped for such work and willing to undertake it, and only carried out by the Committee if no such organizations exist.

In addition the Committee have been entrusted with the Institution's work on specifications and its relations with the British Standards Institution in the very important work of British Standard Specifications.

At the first meeting, the Chairman invited suggestions, both as to the manner in which the work of the Committee should be organized and also for specific problems suitable for immediate attention. As the result of this invitation, a number of valuable memoranda were prepared and considered at subsequent meetings. The following is a brief summary of the Committee's deliberations.

The members were very conscious from the outset of the large number of other organizations which are now concerned with portions of the field of engineering research, and were most anxious that their committee should undertake nothing that would embarrass the good efforts of those already engaged in such work, but rather that means should be sought for co-operation and encouragement. Perhaps, because of the fact that there are so many efforts being made, the Institution's committee may find a special duty in helping to co-ordinate and to publish. It was felt that where problems were considered by the committee as worthy of attack, a definite approach to any other body likely to represent specialist interests should be made suggesting co-operation. Especially was this thought necessary in relations with other and more specialized professional institutions. In fact, the Committee felt that, so far as their work for the Institution was concerned, it was essential to seek the closest possible relations with these specialist groups.

A further general consideration which appeared of great importance to the Committee was the desirability of maintaining a close link with the Local Associations. Many problems likely to arise will require local knowledge, and can be tackled co-operatively between the local group and the main Committee.

It was clear that if The Institution desired seriously to promote research, financial provision for the work would have to be found. This was represented to the Council, who have indicated their willingness to entertain recommendations for expenditure up to a

sum not exceeding £2,000 per annum subject to the qualification that such provision from Institution funds would be dependent to some extent upon financial support being obtained from other sources by the Committee.

It is intended shortly to appoint a full time technical secretary of the Committee, when it will be possible to proceed with further work which is considered desirable. In particular, it is proposed to collect as complete data as possible of the engineering research work at present under way in this country. It is hoped by such a compilation to avoid any danger of the Committee overlapping other work and also to provide a source of information for members, for research workers and for those making a special study of the various branches. It is hoped members will co-operate to make this information service of real value. In the meantime the collection of information from the more easily available sources has been commenced.

With the presentation to the Committee of the various memoranda mentioned earlier, a number of problems for immediate research were suggested, and these were carefully considered by the Committee. As the result, Sub-Committees were appointed to investigate the proposals and the membership has been made much wider than that of the Research Committee itself. The endeavour has been to obtain the assistance of those most likely to have the required knowledge and experience to make the Sub-Committee's work effective, and membership was not restricted to members of The Institution. Some brief notes are appended, outlining the work of the Sub-Committees at present appointed.

Sub-Committee on Wave-Pressures.

At the suggestion of Sir Leopold Savile, consideration was given to the practicability of undertaking experimental work on the measurement of waves on sea-structures.

A Sub-Committee was appointed, consisting of the following members :—

Sir Leopold H. Savile, K.C.B. (*Chairman*).
A. L. Anderson.
G. G. Lynde.
H. H. G. Mitchell, O.B.E.
F. B. Young, O.B.E., D.S.C. (of the Admiralty Research Laboratory).

The Sub-Committee have had several meetings and reported on the following lines :—

(1) That investigations of wave-pressures on sea-structures be made on the breakwater at the Peterhead Harbour of

Refuge and that, for this purpose, a trial single gauge-set for recording wave-pressures electrically, be installed.

(2) That, if the result of the trial experiments obtained from the single gauge-set prove satisfactory, a supplementary programme of research on the basis of the installation of five additional gauges be considered.

These recommendations have been approved by the Committee and forwarded to the Council with a request for an allocation of funds. This has been agreed to, and a sum of £220 made available for the preliminary work. It is hoped that it may be possible to obtain the assistance of the Admiralty Research Laboratory for the design of the gauges and that of the Building Research Station for the analysis of the results. Messrs. Coode, Wilson, Mitchell & Vaughan-Lee have agreed to grant facilities at Peterhead Breakwater and for Mr. W. W. Hill, their resident engineer, to make the necessary instrumental readings and other observations.

Joint Sub-Committee on Vibrated Concrete.

A memorandum was submitted by Mr. Du-Plat-Taylor advocating the investigation of high-frequency vibration of concrete.

A Sub-Commtitee was appointed, under the Chairmanship of Mr. R. H. H. Stanger, to consider the best means of investigating the effect of shaking, percussion and vibrating of concrete during deposition, with particular reference to the effects on setting and ultimate qualities of the concrete.

The Sub-Committee, having learnt that a Committee of the Institution of Structural Engineers had recently been formed to investigate the same question, obtained authority from the Council to invite the co-operation of that Institution, and, as the result, a joint Committee of the two bodies has been established under the Chairmanship of Mr. Stanger, the personnel of which is as follows :—

G. M. Burt.
M. Du-Plat-Taylor.
W. H. Glanville, D.Sc., Ph.D.
W. T. Halcrow.
Oscar Faber, O.B.E., D.Sc.
R. Travers Morgan, M.Eng.
Wm. Muirhead
Gower B. R. Pimm
H. R. Cox
Stanley Vaughan, B.Sc.
W. H. Woodcock
(R. Travers Morgan to W. H. Woodcock) representing the Institution of Structural Engineers.

The Joint Sub-Committee, having given general consideration to the scope of the inquiry, are preparing a comprehensive scheme for

carrying out experiments to determine the effect of vibration on the characteristics (shrinkage, modulus of elasticity, creep, rate of setting, permeability and adhesion) of mortar and concrete (1) during deposition and (2) during hardening.

At the request of the Joint Sub-Committee, the Building Research Station prepared a bibliography of work carried out in other countries and this is now being considered. Individual members of the Sub-Committee have also obtained a quantity of useful information from various sources.

Sub-Committee on the Effect of Soils on Pipes.

Mr. R. G. Hetherington brought forward for consideration the problem of the deterioration of concrete in contact with certain soils.

A Sub-Committee, under his Chairmanship, has been appointed to examine and, as far as practicable, map the occurrence of sulphate salts in the ground, to collect data on the deterioration of concrete, and of iron and steel pipes, in sulphate-bearing grounds and on the remedial measures used, and to arrange, if desirable, for investigations and field tests.

The membership of the Sub-Committee is as follows :—

R. G. Hetherington, C.B., O.B.E., M.A. (*Chairman*).
W. J. E. Binnie, M.A.
Colonel J. R. Davidson, C.M.G.
George Ellson, O.B.E.
C. L. Howard Humphreys, T.D.
F. M. Lea.
R. H. H. Stanger.
John D. Watson.
F. Wilkinson.
A. F. Hallimond, Sc.D. (representing the Geological Survey).
Alexander Melville } representing the Federation of Civil Engineer-
Eric Burt } ing Contractors.

The Sub-Committee have decided to pursue their investigations along the following lines :—

(*a*) To classify the kinds of soil which, through the effect of sulphate salts, are detrimental to concrete, and to ascertain the minimum amount of salt content sufficient to cause deleterious action.

(*b*) To inquire into the best means of protecting concrete from attack.

(*c*) To determine the causes of deterioration of iron and steel pipes due to the same cause.

With this end in view, a questionnaire has been sent to the Engineers and Surveyors of Local Authorities in all parts of the country.

The Sub-Committee will welcome any information of a practical kind which members can give respecting their recent experiences in connection with the location and effect of soils containing sulphate salts on concrete or pipes.

Sub-Committee on Reinforced-Concrete Reservoirs.

The question of the design of reinforced-concrete reservoirs was considered, and a Sub-Committee was appointed to prepare recommendations on the subject of the design of reinforced-concrete structures for the storage of fluids, such as sewage tanks, oil tanks, water towers, etc., and reservoirs of reinforced-concrete construction holding less than 5,000,000 gallons.

The Institution of Municipal and County Engineers, the Institution of Structural Engineers and the Institution of Water Engineers have accepted an invitation to appoint two representatives upon the Sub-Committee.

The membership is as follows:—

W. T. Halcrow (*Chairman*).
Professor Cyril Batho, D.Sc.
W. H. Glanville, D.Sc., Ph.D.
R. G. Hetherington, C.B., O.B.E., M.A.
W. L. Scott.
Sidney Little; F. O. Kirby, M.Sc. } representing the Institution of Municipal and County Engineers.
Oscar Faber, O.B.E., D.Sc.; H. J. Deane, B.E. } representing the Institution of Structural Engineers.
H. J. F. Gourley, M.Eng.; H. C. Ritchie } representing the Institution of Water Engineers.

Sub-Committee on Breathing Apparatus for Use in Sewers, etc.

Professor K. Neville Moss drew attention to the need for more serious consideration of the problem of suitable breathing apparatus for work in sewers and the like.

A Sub-Committee has been appointed, under the Chairmanship of Mr. W. J. E. Binnie, to examine and report on the necessity for prescribing practice for the use of self-contained breathing apparatus for use in sewers, tunnels and similar engineering works.

The Home Office, the Mines Department, the Institution of Municipal and County Engineers, and the Federation of Civil Engineering Contractors have, on the invitation of the Council, each

appointed a representative upon the Sub-Committee, and the membership is as follows :—

W. J. E. Binnie, M.A. (*Chairman*).
R. G. Hetherington, C.B., O.B.E., M.A.
J. B. L. Meek.
Professor K. Neville Moss, O.B.E., M.Sc.
Dr. S. A. Henry (representing the Home Office).
P. L. Collinson, B.Sc. (representing the Mines Department).
E. J. Messent (representing the Institution of Municipal and County Engineers).
G. M. Burt (representing the Federation of Civil Engineering Contractors).

Publications.—If research work is to have general value, it must be widely known as soon as possible, and there is undoubtedly serious need for greater ease of publication than is available at present. Thus the Research Committee welcomed the change in style of the Institution's publications marked by the present issue of the Journal. This should provide the opportunity for much earlier publication than has been possible in the past, and thus help very materially, not only in the work of the Research Committee, but in that of research workers generally.

British Standards Institution.—As noted earlier, the Research Committee has also been asked to assume responsibility for the Institution's work on Standard Specifications. A difficult problem is presented by the representation of professional institutions on such committees as those of the British Standards Institution, for no one man can represent the views of such a large body of members as that of The Institution. On the other hand, it is possible to present effectively the views of a committee, and the Research Committee is obviously the most suitable body to deal with the drawing up of specifications. As already indicated, it is able as at present organized, to ask the assistance of specialists to guide its deliberations, and thus to put forward collective and considered views before any committee with which it co-operates. It is believed that this new arrangement will give really effective assistance to the body of The Institution as well as the British Standards Institution itself.

Future Work.—Whilst the Research Committee has already visualized a considerable field for its activities, it is most sincerely hoped that all members of The Institution will co-operate in this extended activity, and communicate to the Committee suggestions for consideration. Only by such co-operation between all members can the work of the Committee be made of the greatest possible assistance to the whole body of The Institution.

Paper No. 5023.

"Increasing the Water Storage of the Mahadeo Nala Reservoir at Manmad, India."

By George Chappell Minnitt, M. Inst. C.E.

(*Ordered by the Council to be published in abstract form only.*)

Introduction.—Manmad is an important watering station on the Great Indian Peninsula Railway, and it is also the headquarters of the Bridge Engineer of the railway, whose workshops are of considerable size. It is situated on the main Bombay–Delhi line, 162 miles north-east of Bombay.

The main source of water-supply is a reservoir, which was completed in 1911, formed by a masonry dam across the Mahadeo Nala, about 1·3 miles distant from the station. The dam is 712 ft. long and has a maximum height above ground level of 16 ft., which is in the centre portion, where the spillway, 434 ft. long, is formed. It is founded on black trap-rock. The dam was designed for a flood of 3 ft., and possesses a sound margin of safety against uplift, for at no point is the breadth less than 83 per cent. of the height. The catchment-area of the reservoir, which is 13·5 sq. miles, is ample to fill a reservoir of much greater capacity, and the quantity of water originally impounded by the dam was 47,470,000 galls., but by 1927 the capacity had been reduced by silting to 33,250,000 galls.

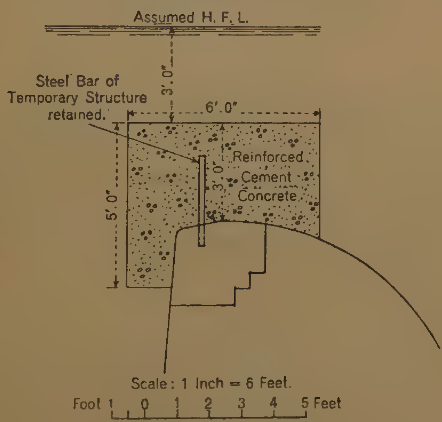
Several schemes were investigated for providing an increased supply, but those most likely from a physical standpoint were rejected on account of their prohibitive cost, and eventually it was decided to raise the crest of the dam in order to provide additional storage.

Temporary Measures.—The profile of the dam was such that the water level could be raised, and it was decided to fix temporary shuttering in position on the crest after the monsoon floods had passed, in order to catch the late rains at the close of the monsoon. Galvanized corrugated-iron sheeting was erected over the length of the spillway during September, 1927, by drilling holes 9 ins. deep by 4 ins. diameter and 5 ft. apart along the cement-concrete crest and fixing therein, with cement mortar, mild steel bars $2\frac{1}{8}$ ins. diameter by 3 ft. long, to which the corrugated-iron sheets were fastened. The whole was rendered water-tight by incising the concrete crest at the junction of the sheeting and the concrete,

and running "dammer," or pitch compound, into the groove to seal the joint; the vertical lap joints of the sheeting were also filled with "dammer." The height of the sheeting was about 2 ft. 3 ins. above the concrete crest, and the spillway was made at one end of the sheeting, where the masonry was already 1 ft. 10 ins. higher than the original spillway. Thus the water level was raised 1 ft. 10 ins. and the additional quantity of water impounded was 17,820,000 galls.

Permanent Arrangement.—After 2 years' observation, it was decided to raise the dam permanently by 3 ft., and at first it was proposed to do this by building a wall of masonry which was to be bonded into the old work with consequent thickening of the profile,

Fig. 1.



without which the line of resultant pressure would be outside the "middle-third." To bond the new work into the old would have meant dismantling the facework and the cement-concrete crest of the dam, and to avoid this, the question of raising the height of the dam without disturbing the masonry was investigated. This was found to be practicable if a mass of cement-concrete were added to the crest, such that the stability of the dam would not be endangered, and in preparing the design of this additional concrete mass, trial and error was adopted to find the most suitable shape and size, the height being fixed at 3 ft. At first a block 3 ft. square was tried, with its upstream face in line with the upstream face of the dam. This was found to be impracticable, as the resultant pressure did not remain entirely within the "middle-third," and eventually the section shown in *Fig. 1* was found to be the most suitable, both from the point of view of construction and of design.

The old masonry was not disturbed, but the cement surface of the concrete crest was chipped and the joints in the masonry were raked out. The additional work is of cement concrete in the proportion of 1 : 2 : 4 by volume, and has simple reinforcement in which the upright bars used for the temporary work are employed. The actual total cost of the permanent work, including a small amount of subsidiary masonry and earthwork, was 20,525 rupees, or 47·29 rupees per foot run, which, at the present rate of exchange, is equivalent to about £3 11*s*.

Since the completion of the work 4 years ago, it has proved to be entirely satisfactory. The extra quantity of water impounded by this additional height of 3 ft. is 28,290,000 galls., which is equal to 60 per cent. of the original capacity of the reservoir, after allowing for evaporation. This is an immense benefit, and was secured at a small cost.

Conclusion.

Either of the methods described is a simple, successful and inexpensive means of increasing the storage capacity of this type of reservoir, although such work would have to be done when the level of the water is sufficiently far below the crest to avoid interference by floods during the progress of the work. The advantages of the permanent method are:

(*a*) The capital cost, and consequently the interest payable thereon, are exceedingly low.
(*b*) There is no recurring charge for maintenance.
(*c*) The method can be applied promptly.
(*d*) The new work can be carried out quickly, without disturbing the existing masonry.

The question of water supply is always of importance, and it is suggested that the methods described may be of assistance to the various authorities when they are considering an increase in the storage capacity of their reservoirs.

Paper No. 5027.

"Sudan Railways, 1925–1935."[1]

By HERBERT DUNCOMBE BINDLEY, M. Inst. C.E.

(*Ordered by the Council to be published without oral discussion.*)

TABLE OF CONTENTS.

IN 1925, Mr. F. G. A. Pinckney, M. Inst. C.E., presented a communication on the "Sudan Government Railways and Steamers."[1] It is the object of this Paper to bring Mr. Pinckney's notes up to date, and to record the developments that have taken place during the last 10 years.

CONSTRUCTION.

Mr. Pinckney wrote when the last railway extension had reached Kassala (Fig. 1, Plate 1). The Sennar dam was opened in January, 1926, and the railway-line was taken across the Blue Nile on the top of the dam to a terminus at Kassab El Doleib, which became also the terminus of the steamer-services plying on that river up to Roseires.

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[1] Inst. C.E. Selected Engineering Paper No. 35.

Kassala-Gedaref Railhead.—It then became clear that the next extension offering the best prospects of an immediate return in the form of new traffic-development was that from Kassala to Gedaref. This extension necessitated the construction of a large bridge over the river Atbara, near Khashm El Girba, and the work was therefore divided into two portions, namely, Kassala to the river Atbara (68 kilometres), and from the river Atbara to Gedaref (150 kilometres).

In the spring of 1926, arrangements were made at Kassala to provide for a depot for storage of permanent-way materials and stores. Railhead-construction started on the 14th December, 1926, and progressed at the rate of a mile a day, reaching the river Atbara on the 21st January, 1927. This section of 68 kilometres runs through belts of "kitter bush" over gently undulating ground, and presented no engineering difficulties until the "kerrib" was reached at the 58th kilometre from Kassala. This curious formation of miniature hills and valleys made the approaches to the bridge very difficult to negotiate, involving high banks and deep cuttings in order to avoid exceeding the maximum ruling gradient of 0·66 per cent.

As soon as the river had fallen sufficiently in the autumn of 1927, Messrs. Dorman, Long & Company, the contractors for the bridge, were able, by means of a floating pile-driver, to complete the construction of a temporary bridge across the river. This consisted of sixteen spans of 15-foot standard bridge-steelwork carried on bents of wooden piles driven four abreast. Some difficulty was experienced in getting the piles down to the depth required, as the rock was found to be harder than had been anticipated. The difficulty was overcome by the use of rock-points. Subsequently, some of the bents settled under the impact of the moving trains over the bridge, but, by careful inspection and packing, stability was attained a month after the bridge had been opened to traffic. The structure was completed on the 24th November, 1927, a week ahead of scheduled time, and the second portion of the line-extension to Gedaref was put in hand.

The labour-force for the construction of the line began to arrive on the 26th November, and railhead was carried forward to the top of the "kerrib" on the western bank of the river. Thereafter, progress on railhead was increased gradually until the daily rate of 1½ miles was attained. Gedaref was reached on the 18th February, 1928, whereupon the whole labour-force was transferred to Kassab El Doleib, and 34 kilometres of track was laid to Es Suki, on the Blue Nile, and from thence towards the Dinder river. This work was carried out with a view to getting materials to the bridge-site

on the river Dinder, and thus expediting the work and the final opening of the line from Gedaref to Sennar, which was scheduled to be carried out during the winter of 1928–29.

Butana Bridge.—Meanwhile, work on the construction of the Butana bridge over the river Atbara had been progressing. Work was actually started by the contractors early in December, 1927, on erection of the caissons, starting from the east bank and working out across the river as it fell. Where it was not possible to place the shoe in the dry, an earth embankment was built in the river, which, by that time, had ceased to have any appreciable flow.

The caissons were sunk by means of compressed air to the contract depth, with the exception of No. 3, at the site of which hard rock was met at a higher level than had been indicated by the exploratory borings. On completion of two of the piers the steel-work of one span was erected on staging and, thereafter, each span was built out without staging, a temporary connection being made between each truss to provide the necessary support. The bridge, comprising seven spans of 150 feet, was completed on the 9th July, 1928.

The river had come down in flood in May, and the temporary bridge had been removed early that month in anticipation of it.

Dinder Bridge.—It was planned to construct the Dinder river bridge first, working from the railhead east of the Sennar dam, and, as soon as rail-communication was through from Gedaref, the Rahad bridge was to be undertaken and completed before the river came down in flood in June. The Rahad was to be crossed by railhead in the first instance by means of a diversion-bank through the river, which dries up into a series of pools during the dry season.

On survey, the site of the Dinder bridge was first selected on a straight. The depth of sand liable to scour and the waterlogged substratum, as shown by borings, led to a further study being made of the siting. Reconnaissance showed that, near bends in the river, water was held in pools, which indicated a substratum of clay; in addition, the banks showed clayey bands. A detailed survey was made of further possible sites and, as a result, a line of bores was put down, the results of which proved to be satisfactory. The bridge is sited on a short straight between two sharp bends, where the ground on the approaches was considerably higher and the channel narrower than at the site first selected; only three spans of 105 feet were required as against four on the previous line.

The period available between the fall and rise of the flood was 6 months, namely, December to May inclusive. Railhead from Es Suki reached the bridge-site on the 5th November, 1928. Con-

struction-sidings were laid on the west bank, cranes and materials brought as near to the site as river-conditions would permit, and work on the construction of the west abutment put in hand forthwith. This foundation was built in an open timbered trench, the excavation being for the most part through dry sand, which gave more trouble than had been anticipated. It was not found possible to excavate ahead of the close sheeting, which was driven from the top as far ahead of the excavation as possible. At times, difficulty was experienced in keeping the sheeting from travelling inwards as it was driven. By the application of pressure by means of hydraulic jacks through heavy struts it was found possible to correct this tendency. Hard clay was reached and penetrated to a depth of 5 feet, and the concrete foundation laid in this. The masonry was then built as quickly as possible, as continual leakage of dry sand through the timbering gave much cause for concern.

The river was late in falling, and it was not until the 11th December that a causeway could be built out to the west pier. By the time the caisson-shoe had been placed and riveted, a siding had been laid down into the river-bed and steam cranes had been erected each side of the pier site. Each crane was fitted with a $\frac{3}{4}$-cubic yard sand-grab.

When the shoe had been sunk to a depth of 6 feet, the brickwork was built upon it to a height of 6 feet above ground-level, and grabbing was resumed. Grabbing became slower after reaching the harder underlying clay. After the concrete slab had been cast, a shell of masonry was built up as the work proceeded, leaving the largest possible dredging-holes, but forming the cutwaters at each end. In view of possible scouring this was considered better policy than carrying up the square walls.

After penetrating the clay, the grabs removed as much material as possible, but it was found necessary to excavate by hand under the cutting-edges of the shoe. A 6-inch steam pump was able to keep down the water in the caisson to a level at which the men could work effectively. After the caisson had been founded, the wells were pumped dry and sealed off with 4 feet of concrete. The wells were then filled with concrete to the top of the shell and the pier completed in rubble-granite masonry in cement mortar. Rapid-hardening cement was used for filling between the brick facings of the caisson-walls and also in the mortar for the masonry of the shell. With the use of this cement and of steel-mesh reinforcement in continuous lengths, little time was lost between building and sinking, and at no time were any signs of cracking observed. The second pier was constructed in a similar manner.

It had been intended to excavate an open timbered trench for

the foundation of the east abutment, as was done on the west bank, but, after 18 feet had been excavated through alluvium, wet sand was disclosed. A trial hole was excavated further and water was met with. It was therefore decided to use a caisson for this abutment similar to that for the piers, and the work was carried out accordingly. The foundation for the land-abutment for the 30-foot approach-span was taken out in open trench to a depth of 8 feet. Two wells were then sunk to a further depth of 23 feet. These were filled with concrete and connected to the main foundation by steel bonds made of old rails. The wells were sunk to prevent slipping of the foundation should the strata become waterlogged during the flood-season. The depth to which the wells were sunk was determined by taking a slope of 2 to 1 from the river-bed level at the east pier.

The steelwork consists of three spans of 105 feet and one approach-span of 30 feet. The 105-foot span is a standard, and two of these spans were available in stock. Expansion of the use of motor-transport by the Government made it desirable to provide a roadway over the bridge, but the spans in stock could only be utilized if strengthened to take the additional dead load. This was carried out, and one of the spans was used on this bridge with two new spans of 105 feet and one new span of 30 feet. The erection of the steelwork calls for no special comment, but the design of the false-work is of interest. The trestles were made by native carpenters by mass-production methods, and were to a standard design as far as possible. The top stage was formed of trestles 18 feet long, and the bottom stage of trestles of definite lengths, which could be put on sills at determined levels.

The bridge was completed and the first train passed over on the 17th May, 1929, and the river came down in flood, 10 feet high, on the 23rd May.

Rahad Bridge.—The first survey of this bridge-site disclosed fairly extensive swamps to the west of the river. The approaches were kept to the only high ground available, but at the river itself a permanent pool, about 13 feet deep, existed all through the dry weather. The river-bed consists of a series of pools only connected when the river is in flood. Borings carried out by a hand-operated rig disclosed permeable strata, and anticipated constructional difficulties at this site led to a further study being made, which resulted in the decision that the most suitable site would be one which contained no pool, or, at most, a pool of shallow depth. It had to be borne in mind that the railhead-construction party working from Gedaref would require a nearly dry river-bed to enable it to cross on a diversion not too far distant from the bridge-site. To construct

and maintain a crossing in a deep pool presented many difficulties and an element of danger, and would inevitably have caused serious delay to construction.

Survey-work was therefore resumed at the river and a site located where the approaches, although necessarily on a high bank, could be made in the dry, and where the river-pool was only 3 feet deep. Exploratory bores were put down with a hand-operated rig on the line of this new site and found hard clay at a moderate depth.

As the dredging-cranes became available from the Dinder bridge, they were sent up to the site of the Rahad bridge, this having been made possible by the completion of railhead-construction on the 15th February, 1929, and the linking-up of the line at a point east of the Dinder bridge, reported on p. 55.

The first caisson was placed for the east pier and was sunk by similar methods to those used on the Dinder bridge. The caisson-shoe for the west pier was delayed in transport, and an attempt was made to found the pier by a timbered trench. When within 4 feet of the prescribed depth, heavy rain fell, causing the clay to swell and burst the strutting, which resulted in dangerous caving in of the sides. Whilst ineffectual efforts were being made to repair the damage, the caisson-shoe arrived. The excavation was partially filled in, the caisson-shoe placed and grabbing operations started, and the caisson was founded without further incident. The two abutments were built concurrently with the east pier.

Trestles for the erection of the steelwork were available from the Dinder bridge, and the same general methods of erection were followed.

The bridge, consisting of one span of 150 feet and two of 55 feet each, was completed on the 9th June, 1929, and the river came down in flood on the next day.

Gedaref—Dinder Railhead.—For the extension from Gedaref to the Dinder river, the same base depot at Kassala was used for the stacking of permanent-way materials and stores.

Prior to the starting of railhead-construction from Gedaref, part of the labour-force was collected at Es Suki and laid 16 kilometres of track from the railhead left there in April to the Dinder river, to allow work on the construction of the bridge to proceed. This labour-force was then transferred to Gedaref and augmented to make up a force sufficient to construct the embankment and lay $1\frac{1}{2}$ miles of track per day, which was begun on the 1st December, 1928.

From Gedaref, the line runs over slightly undulating and very open grass-country for the first 60 kilometres, at which point the first small gum-forest is met, and the line then turns among the small

hills to Qala En Nahl. From that station the line falls on a gentle gradient all the way to the river Rahad, passing through gum-forests with sections of thick kitter bush at intervals. After crossing the river Rahad the line, running north of Khor El Atshan, falls gently all the way to the river Dinder. The first 10 kilometres west of the Rahad are through gum-forest and occasional kitter bush. This is followed by about 25 kilometres of open grass-land with scattered trees—the typical game-country of Africa. For the last 20 kilometres to the river Dinder the line passes mainly through kitter bush of varying denseness with occasional open spaces, but getting more dense as the river is approached.

The soil throughout is cotton soil, being fairly heavy near Gedaref but lighter and more friable through the gum-forest and kitter-bush areas, and very black and very heavy on the section between the Rahad and Dinder rivers.

Whilst railhead was approaching the Dinder river from the east, the construction of the bridge over the Dinder was being carried out, a temporary diversion was constructed on a bank across the river-bed, and the track was laid as far as a point about 1 kilometre east of the river. Railhead from Gedaref reached this point on the 15th February, 1929, thus completing the line through from Haiya, Kassala and Gedaref to Sennar, and affording an alternative route to Port Sudan for the produce of the central Sudan.

Following up behind railhead there was a building party for the construction of station-buildings and quarters, and also a bridge-building party. Quarters and buildings were kept down, in the first instance, to minimum requirements, and were for the most part of the "tukl" type, being an adaption in bricks and mortar of the typical native huts of the central and southern Sudan, where the usual materials of construction are mud or reed walls with grass thatching for the roof.

Bridges were sited during railhead-construction and details sent back to the bridge-building party; two small survey parties were formed immediately after completion of the tracklaying to run cross sections and make contour surveys to determine water-shed areas, etc., in connection with some of the bridges and pipe-culverts.

Between Kassala and Gedaref, water-stations were easily obtained where the railway approached the river Atbara. In Gedaref, water in limited quantity, but containing nitrites, was obtained from an open well at a depth of 47 feet in decomposed basalt. A series of deep bores was put down to ascertain if a better supply could be obtained from the lower rocks. One bore was drilled to 294 feet entirely through basalt and, though the additional supply given

was small, the water was free from nitrites. At Qala En Nahl water in small quantity was found at 136 feet, and this was made a locomotive-watering station. In spite of the proximity of the river Rahad, no underground water was found by boring at the stations on either side of that river. Borings at Khor El Atshan, nearer the river Dinder, gave water at 115 feet depth, and this station was made the next locomotive-watering station, further supplies being obtained at Es Suki by pumping direct from the Blue Nile.

The track throughout was flat-bottom rail (50 lbs. per yard) on steel pea-pod sleepers, using rails of 30-foot length with twelve sleepers per rail-length. The practice of laying track with staggered joints [1] was abandoned in favour of square joints except on curves of 2 degrees and over, which are still laid with staggered joints to facilitate maintenance of good alignment. The change of practice was brought about after completion of the extension to Gedaref, as it was found that, in this area of considerably heavier annual rainfall, not only was track with staggered joints more difficult to maintain but, as considerable sinkage of track occurred at times during the rains, undue stresses were liable to be caused in the underframes of rolling stock. Moreover, the occurrence of extremely low joints increased the liability to derailments due to oscillations set up. The wheel-base of the average 30-ton bogie goods-truck is 23 feet from centre to centre of bogies, giving about 18 feet 4 inches from centre to centre of the inside wheels.

Mr. Pinckney points out [2] that during the rains on the earth-ballasted track south of Khartoum the track must of necessity be left alone as far as possible. This is accentuated on the Kassala—Gedaref and Gedaref—Dinder sections, where the annual rainfall is considerably heavier and the cotton soil of a heavier nature, rendering the lifting and packing of the track an impossibility during the rains, and therefore necessitating the adoption of the type of track-laying best suited to meet these special conditions.

Flood-conditions in 1929 were severe, and the river Rahad overflowed its west bank, throwing a large volume of water against the railway-bank, which had no bridge-openings to let the water through. This flood-water rose above rail-level and flowed over the bank to a depth of 18 inches for a length of approximately ½ kilometre. An extensive survey was made upstream of the bridge-site in an endeavour to locate any large spill. Results indicated that, although no large spill existed, the river had overtopped the west bank for long distances by 1 foot. The spill had flowed westwards until it

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[1] *Ibid.* p. 54.
[2] *Ibid.* p. 57.

reached the first depression and, forming itself into a subsidiary river, flowed northwards across the railway. To pass this water in future years the railway-bank was raised 2 feet above the flood-level, and four bridges were built to pass the spill-water under the line. So far these measures have proved completely effective, and no further trouble has occurred on this account although at least two floods have been even higher than that of 1929.

Port Development.—During the 4 years that railway-construction was being carried out considerable developments were being undertaken at Port Sudan.

At the time Mr. Pinckney wrote in 1925, work on the construction of two new berths for coal-ships on the west side of Port Sudan harbour had been commenced. The work consisted of a quay-wall, 900 feet long, in concrete blocks, reclamation behind this, and the construction of mass-concrete walls for transporter-tracks. This work was completed during 1926, when work on the north extension of the main quays was undertaken. Some delay occurred in this latter work owing to the insecure foundation of the existing quay, which prevented a junction of the new with the old work. After investigation it was decided that at least 100 feet of the old quay would have to be taken down and rebuilt. This reconstruction and the new extension, giving 230 feet of new quay-wall, were completed in 1928. Meanwhile, surveys had been carried out and plans prepared for the construction of an oil-berth, 450 feet long, which was completed in 1928, and for a salt-berth, 541 feet long, which was begun in 1929 and completed in 1930.

In 1929, an additional slipway was constructed at the dockyard.

Other Line-Extensions.—Owing to the universal trade depression, resulting in the curtailment of capital works, retrenchment and the necessity for drastic economies on maintenance-expenditure, further developments since 1929 have necessarily been confined to minor works of urgent importance only. Nevertheless, the period has not been without some construction-work, due to external influences.

The following line-extensions have been carried out:—

(*a*) 8 kilometres of track from Khartoum to Gordon's Tree, the site for the new dockyard of the Egyptian Irrigation Department.

(*b*) 38½ kilometres of track from Gordon's Tree to Jebel Aulia, the site of the new dam across the White Nile which is being constructed by the Egyptian Government in connection with increasing the water-supply for irrigation in Egypt, this dam being a sequel to the raising of the Aswân dam.

(*c*) 8 kilometres of track from a point 30 kilometres north of Khartoum to Jebel Sileitat, the quarry-site for granite stone for the construction of the Jebel Aulia dam.

All these lines were constructed in accordance with the usual out-of-face method of construction utilized on previous railhead-constructions.

With the exception of the first 5 kilometres of the line to Jebel Sileitat, 50-lb. flat-bottomed rails on steel pea-pod sleepers, twelve per 30-foot rail, was laid. The first 5 kilometres of the line to Jebel Sileitat, that is, from the junction on the main line to the station at the quarry, where full trains are taken over from the contractors and empties handed back, the track was laid with 75-lb. flat-bottomed rails with fourteen sleepers per 36-foot rail. This was done to allow of the heavy main-line engines running right up to the quarry station.

Circumstances, however, caused two main alterations to be made in the usual arrangements. The first was the employment exclusively of native labour of the Sudan for all the work. Previously it had been the custom to engage Egyptian Saidis from Upper Egypt (between Luxor and Shellal) for all the heavy work of constructing the embankment and rail-carrying. The second was the provision of water for bankhead parties. On previous occasions, all water had to be brought up to railhead by wheeled tank and distributed from there by camels, each camel carrying two 15-gallon galvanized-iron water-tanks. The parties on bankhead might be from 1 to 3 miles, or even more, from railhead. The Saidi is a bigger, stronger man than the average native of the northern Sudan and, having been accustomed to hard agricultural and manual labour all his life, he is able to carry out a larger task on earthwork, so that he requires less water per cubic yard of finished bank than the native of the Sudan, which is a big consideration when every drop of water has to be transported for such long distances.

In the case of the construction to Jebel Aulia, the line was never very far away from the river and it was therefore possible to supply the whole of the bankhead party, by means of camel-transport, with water direct from the White Nile. In these circumstances there was no special reason to bring in labour from Egypt, and all that was necessary was to start the bankhead party well ahead of railhead tracklaying; this was so timed that the earthwork, including the layout for the terminal station at Jebel Aulia, was completed the day before the tracklaying party completed their work.

Buildings.—The construction of additional offices, quarters, etc., followed the development of line-extensions, but no very large buildings were erected during the period under review. New

foundations were constructed for an extension of the Atbara power-station. The concrete foundation blocks were 30 feet by 11 feet by 9 feet, one of them being cast on a caisson sunk 29 feet down to solid rock. The other was cast on three wells also sunk to the same depth to rock. The triple-well type of construction was adopted in the one case on account of the close proximity of the new foundation to the wall of the existing power-house and to an old existing engine-foundation, which allowed of considerable vibration when the engine was running. The old foundation has recently been broken up and replaced by a new block, cast on a caisson sunk to solid rock. The method of carrying out the work was similar to that employed on the first two foundations, but the conditions were very much more difficult, as the work had to be done inside the power-house whilst the latter was in use.

Continuous study has been given to improved and modern methods of construction, more especially as regards foundations and roofs and in the use of steel-frame doors and windows, precast-concrete verandah-posts, steps, lintels, etc. Special study has also been given to methods for combating termites with a view to making buildings termite-proof.

Maintenance.

Bridges.—The total number of bridges on the system is as follows :—

(*A.*) *Large Bridges.*

(i) Over Atbara river at Atbara ; seven spans of 147 feet.
(ii) Over Blue Nile at Khartoum ; seven spans of 218 feet 6 inches and rolling-lift span of 111 feet.
(iii) Over White Nile at Hillet Abbas ; eight spans of 155 feet and swing-span of 242 feet.
(iv) Rolling-lift span of 115 feet at Port Sudan.
(v) Over river Atbara at Butana ; seven spans of 150 feet.

(*B.*) *Bridges of Standard Spans.*

Span: feet.	Number.
105	85
55—16	169
15 and under	2,507
Pipe-culverts	1,718

Pipe-culverts are constructed of 3-foot and 2-foot diameter cast-iron spigot-and-socket pipes ; 3-foot diameter corrugated-iron pipes ; 2-foot diameter reinforced-concrete pipes, manufactured in the country ; 2-foot 3-inch, 2-foot, and 1-foot 9-inch diameter steel pipes ; and a few 4-foot and 5-foot diameter culverts made from old locomotive-boilers.

The standard 15-foot bridge-span, which predominates, has been used in multiples up to nineteen spans. This span has been a very handy size for handling in the field with native labour, thereby facilitating erection, and freight and transport costs are considerably lower than they would be on the steelwork for larger spans. It has been found, however, that considerable trouble is experienced with these bridge-spans on cotton soil. Generally speaking, difficulties in ordinary soil arise from subsidence, but in cotton soil troubles occur through the piers of the bridges composed of two or more spans rising owing to the expansion of the soil after rain. The piers are not as heavy as the masonry abutments, and not only do they rise but also get out of alignment. To rectify this, it is necessary to insert hardwood packings under the bearing plates to correct the levels, and then to re-align the track by re-boring the sleepers. Although it is not a difficult matter to adjust the levels each year by inserting packings of different thicknesses, which are always kept ready in stock, it involves a good deal of work and careful levelling. Additional trouble and expense is, however, involved in re-aligning the track, and the sleepers soon become "spike-killed" and have to be renewed long before the life of the rest of the sleeper is expended.

To eliminate these troubles, a trough span was designed in 1928 and used on the extension from Gedaref to Dinder. This type being a stone-ballasted bridge, any alteration in level or alignment caused by the movement of the piers can be adjusted readily by the ganger in the course of his work by ordinary methods of lifting and packing and slewing of the track.

Whenever it has been found necessary to add additional spans to an existing bridge on cotton soil, the opportunity has been taken of reconstructing the existing abutments and piers to suit the new trough steelwork, and the old type of box girder has been re-used elsewhere where solid foundations were obtainable.

Track.—In January, 1935, the total route-kilometrage was 3,253, and track-kilometrage 3,604.

The organization, described by Mr. Pinckney, of having a native headman in charge of each section-gang with more experienced gangers in charge of three or four stations over the headmen has been gradually replaced by the method of having a native headman in charge of each station directly under the track-supervisors, whose average length for supervision is ten stations, or 240 kilometres. This has been attained by careful selection of men for promotion and for new appointments, by the training given to the headmen by the British permanent-way inspectors, and by holding periodic examinations under the chief permanent-way inspector at headquarters. By this means, a higher standard has been attained,

and it has been found desirable and more economical that each headman should have full charge and responsibility for the maintenance of his section directly under the section track-supervisor.

Under the new organization of the engineering department, introduced towards the end of 1934, the railway-system is divided into four districts each with two British engineers. The districts are divided, according to their length, into four or more sections, each with a track-supervisor in charge, who may be a British permanent-way inspector or a native assistant superintendent of ways and works. Each district has a native district surveyor and, in those districts with the larger towns included, a British superintendent of works, with a native assistant surveyor under training, are added for supervision of maintenance of buildings and quarters in the towns.

The native district surveyors and assistant superintendents of ways and works have risen from their first appointments as assistant surveyors and have, under the guidance of the British engineers under whom they have worked, become far more reliable in their work and more accustomed to the outdoor life of railway-work. Special classes are held annually for all those employed on track-work and examinations are held on permanent way and general rules and regulations after each class of instruction.

Commencing in 1928, the issue of a permanent-way manual, standardizing the methods and practices, etc., for the maintenance of track, was put in hand and was issued, in the first instance, in typescript. The manual was written as a guide to district engineers and track-supervisors, as a help to newly appointed officials, and, above all, as a record of the approved standards to be followed. After having been in use for several years in this form and subject to amendments from time to time, the work has now been completed and printed, and issued to all English-speaking staff engaged on track-maintenance.

Following the practice of many railways and the recommendations of the International Railway Congress of 1930, the question of equation of track for maintenance was given careful study, and a formula evolved. The study was completed towards the end of 1930, and the formula was immediately adopted as the basis for the reduction of the strength of section gangs that became necessary to meet the requirements of immediate economy called for by the world depression in trade. After 4 years of study and working to the lowest labour figure possible under the adopted formula, it has been found that small arrears of maintenance, not particularly noticeable at the time, have accumulated from year to year. The equation has been the subject of recent revision, and a number of

gangs have been increased in strength. In order to ensure that the revised formula may, in future, be compared with normal conditions, the accumulated arrears of maintenance have been made up as far as possible by small special parties working for a few months during that part of the year most favourable to the execution of the work.

The main improvements in track-materials during the period under review have been the adoption of a new type of fishplate (*Figs. 2*) for the 75-lb. rails, to replace the older types of flat and angled fishplates. The new design follows very closely the recommendations in the Sixth Progress Report of the Special Committee of the American Railway Engineering Association on Stresses in Railroad Track.[1]

Following the home-railway practice, from 1934 the 75-lb. rails have been ordered to the higher-manganese specification adopted by the London and North Eastern Railway.

A new type of bearing plate (*Figs. 2*), mainly for use on bridges, was adopted in 1934. By having the rail held to the bearing plate by a clip and bolt and utilizing the screw-spike only to hold the bearing plate to the sleeper, it is anticipated that increased life of bridge-sleepers and bridge-timbers will be attained, apart from the increased bearing-area which this type of bearing plate affords.

Relaying with 75-lb. track in place of 50-lb. track has been commenced on the Khartoum—Wad Medani section, which will eventually allow of heavier engines and longer trains being run over this length, which carries a fairly heavy traffic during the cotton season. Sixteen worn crossings have been reconditioned by electric welding, and additional plant is being purchased to enable this work to be carried out on a larger scale during future years.

Study has been given to the wear of rails on curves. Under normal conditions the average life of rails in this country is placed at 60 years. On some of the 4-degree curves on the Port Sudan line, 54 per cent. of the allowable loss in the head of the rail has occurred in just under 4 years. Four rail-and-flange lubricators, using graphite grease, have been installed, and the reductions in the rates of wear of rails and locomotive-tires are being watched.

Attention is being given to the elimination or anchoring of sand-dunes, which threaten to cover the railway-track over one section of the line, and drift-sand also causes considerable blocking-up of bridge and culvert openings on other sections. A very full summary of the experiences of many railways was published by the American Railway Engineering Association,[2] and some of the experiments

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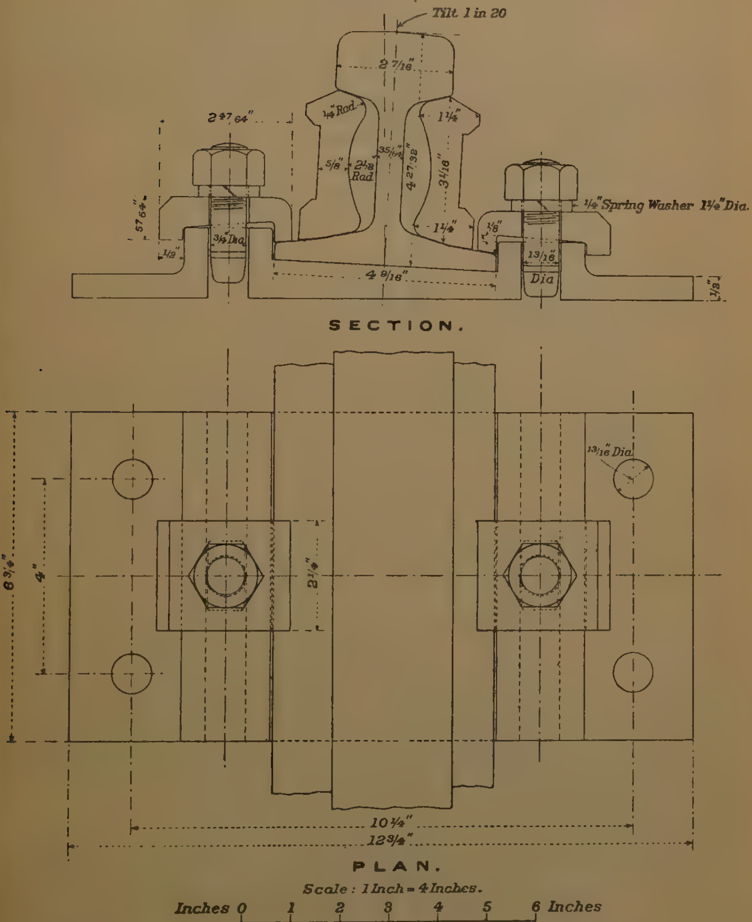
[1] Bulletin No. 358, vol. 35 (1934), Chicago, p. 69.
[2] Proc. Am. Ry. Eng. Assoc. Vol. 35 (1934), p. 376.

recorded therein are being adapted to local conditions and tried out. It is estimated that the annual cost of keeping the one length of 35 kilometres of track free from sand for the safe passage of trains amounts to approximately £700 per annum.

In order to increase the development of the natural timber

Figs. 2.



resources of the country, renewed efforts were made by the Chief Conservator of Forests in 1931 to induce the railways to make use of Sudan mahogany sleepers, which can be obtained from the Bahr El Ghazal province in the south. After several conversations and a full investigation into the reasons for the failure of this timber in

the past, the Forestry Department started a saw-mill near Wau in 1932. Arrangements were made for all sleepers to be sawn to size, due allowance being made for shrinkage, and kept in the damper atmosphere of the Bahr El Ghazal province to season for 2 years before being brought up north for creosoting and use in the track. The first consignment of these sleepers was received at the end of 1934 and is being put in the track this year. The results, as regards freedom from warping and serious cracks and splits, have been satisfactory, and are a very great improvement on those of former attempts made to bring up logs and cut them to size after arrival up north. Further trial is to be made with "sunt" wood, of which small forests exist along the banks of the Blue Nile, and the Forestry Department are expecting a regular supply of 70,000 seasoned "sunt" sleepers to the railways per annum, commencing in 1937.

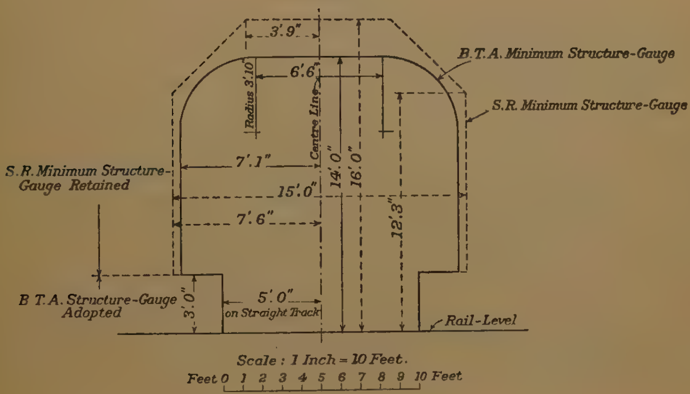
Re-timbering of three of the large bridges, namely, those over the White Nile, Blue Nile, and Atbara, was undertaken at the end of 1934 and early in 1935. These bridges were originally timbered with jarrah timbers, 8 feet by 10 inches by 8 inches, in 1910–11. They were laid with 8-inch by 8-inch longitudinal guard-timbers outside the rails, but, consequent upon the adoption of the British Tropical African structure-gauge, it was decided that when the bridges were re-timbered they should be fitted with internal guard-rails in place of the external guard-timbers. One of this Railway's delegates to the International Railway Congress in Spain in 1930 made a study of the guard-rails in use on the Madrid, Saragossa and Alicante Railway in that country, and, from plans supplied by that Railway, details of the fixing of the new guard-rails, together with re-railing devices for each end of the bridges, were designed (Fig. 3, Plate 1), and the necessary work was carried out in the railway workshops in Atbara. A trial re-railing device was first tested in the track, derailed waggons being drawn over it at various speeds. Following on the adoption of internal guard rails in place of the external timbers it has been possible to reduce the length of the bridge timbers to 7 feet.

The particulars and conditions respecting stone ballasting of track remain much the same as described by Mr. Pinckney on pp. 8 and 9 of his Paper. A length of cotton-soil bank near the Rahad river was ballasted with stone bottoming and laterite ballast, and another section with laterite ballast only, but neither have proved very satisfactory owing to the percolation of water into the cotton soil below; further, the laterite is not of first quality and becomes "mushy" during the rains. More success has attended efforts at filling over the cotton soil with disintegrated

granite, which forms the overburden in a small stone-quarry opened in 1928 near Gedaref. This disintegrated granite forms a hard waterproof surface from which the rain runs off as it falls and, as no percolation into the underlying cotton soil takes place, the condition of the track during the rains is vastly improved. Moreover, as this material dries out comparatively quickly, it allows the section-gang to get to work on the lifting and packing of the track some time before similar conditions are attained where cotton soil is used as earth ballast.

Water-Supply.—A chemical laboratory was constructed and opened in Atbara in 1925. A regular monthly examination of all the waters in use on the railway was made; in addition, special

Fig. 4.



analyses were made of the water produced during the sinking of new bores, and proved of great value.

Investigations were made into various matters of importance, such as the causes and prevention of priming, scaling and corrosion in locomotive-boilers. Experiments were carried out with small boilers working under high pressure (160 lbs. per square inch), and the results of these experiments were tried out with locomotives in service. Water-softening plants were installed with satisfactory results at Port Sudan, Kamob Sanha, and Gebeit as a result of experiments that demonstrated very clearly that economy could be effected both in coal-consumption and boiler-repairs by treating all water for use in locomotive-boilers on the Port Sudan line.

The demands for economy in 1931 resulted in the special laboratory being closed down, and such analyses as are now required are carried out by the Wellcome Research Laboratories in Khartoum.

Standard Loading- and Structure-Gauges.—No alteration has taken place in the standard loading-gauge but, in 1931, on representation from the Crown Agents for the Colonies, the proposed structure-gauge for British Tropical Africa was adopted in part, that is, for those dimensions that were greater (measured from the centre line of the track) than those already in use (*Fig. 4*). The alterations were adopted for all new structures as from 1932.

Buildings.—In conjunction with the introduction of modern methods of construction, study has also been given to improved maintenance, both in methods and materials, and, until 1931, large annual-renewal programmes were being carried out with a view to progressive replacement of old buildings on which maintenance-costs were heavy. Particular attention has been paid to selecting suitable kinds of paints for both external and internal use. Large replacements have been made of timber door- and window-frames by steel frames, manufactured in the railway-shops, and of wooden shelving by precast-concrete shelves, in cases where termites have been active.

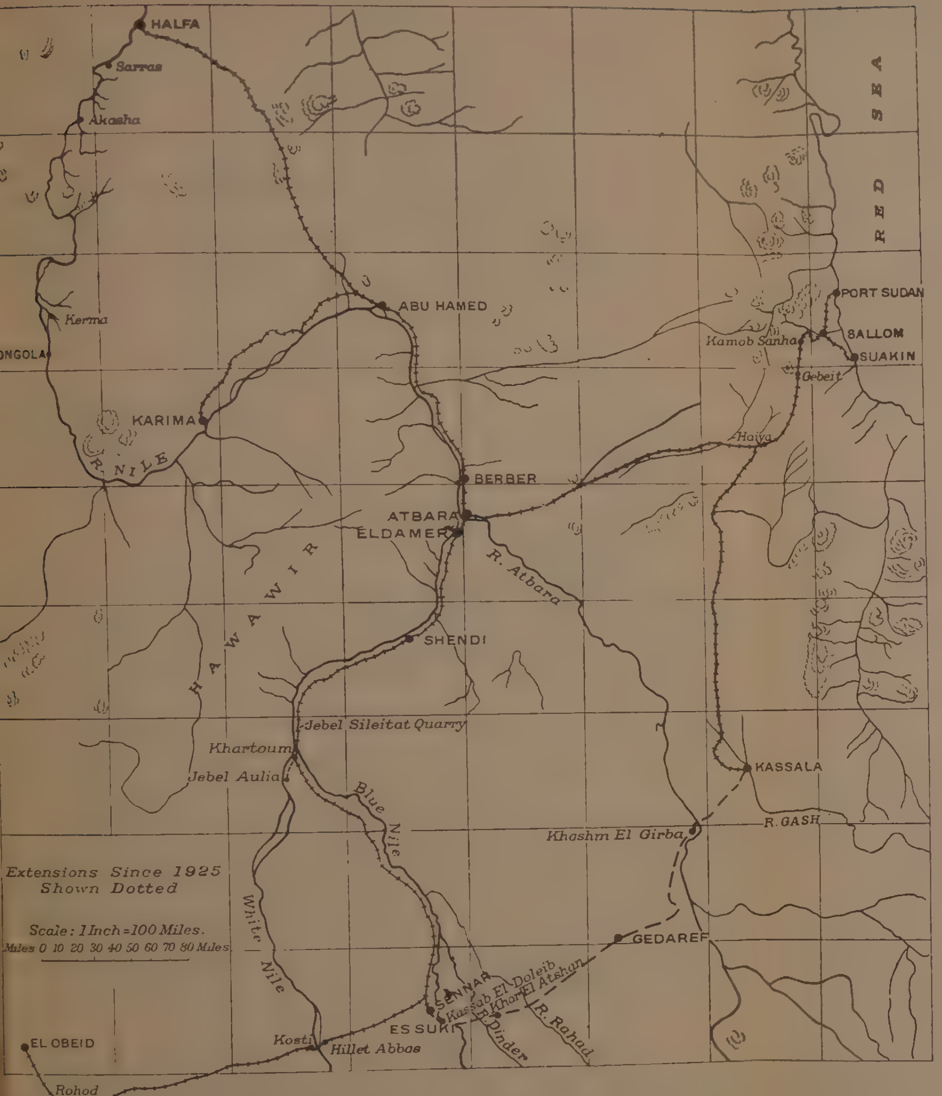
Reduction of staff on retrenchment and the reorganization of various departments in recent years has resulted in surplus quarters and buildings at some stations. A number of such quarters and buildings have been demolished in cases where it could be seen that they would not be likely ever to be required again.

The Paper is accompanied by a map and fourteen sheets of drawings, from some of which Plate 1 and the Figures in the text have been prepared.

The Council invite written communications on the foregoing Paper, which should be submitted not later than three months after the date of publication. Provided that there is a satisfactory response to this invitation it is proposed, in due course, to consider the question of publishing such communications, together with the Author's reply.

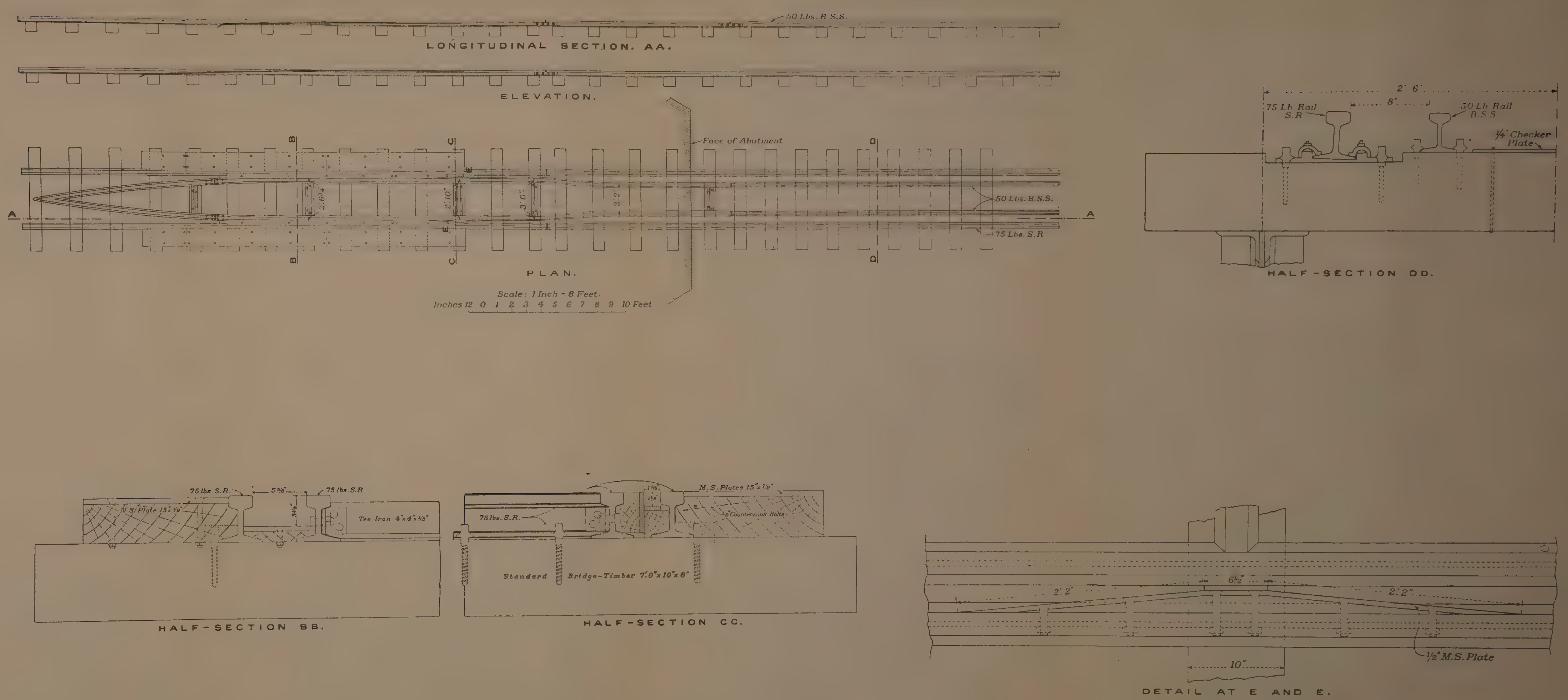
SUDAN RAILWAYS, 1925–1935.

Fig: 1.



THE SUDAN RAILWAYS.

Figs. 3.



Scale: 1 Inch = 1 Foot.
Inches 6 5 4 3 2 1 0 1 Foot

GUARD-RAILS.

H. D. BINDLEY.

Waterlow & Sons Limited, late Thos. Kell & Son, London.

Paper No. 5003.

"The Open-Frame Girder."

By Edward Hugh Bateman, M.A., B.Sc., Assoc. M. Inst. C.E.

(*Ordered by the Council to be published without oral discussion.*)

TABLE OF CONTENTS.

Introduction.

The girder without diagonals is a recognized method of construction, especially in reinforced concrete or welded steelwork, and the principle of the open frame is also of considerable importance when applied as a component of the principal system, for example, as the rib of an arch or as the stiffening girder of a rigid suspension-bridge.

The problem of the stress analysis of this system has attracted the attention of a number of writers, and Dr. Ing. Karl Kriso gives a long investigation and an extensive bibliography.[1]

The method of analysis used in this Paper has been enunciated by the Author in two recent Papers,[2] and has already been used to solve a similar problem.[3] This method gives a shorter and simpler derivation of the elastic equations and enables the solutions to be determined in a generalized form.

The greater part of the Paper is concerned with the detailed analysis of what may conveniently be called the girder of uniform

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[1] "Statik der Vierendeeltraeger," Berlin, 1922.

[2] (*a*) "The Strain Energy of an Elastic Bar: a useful Transformation," Phil. Mag., vol. xvi (1933), p. 1128.

(*b*) "The Strain Energy Method in Elastic Network Analysis, illustrated by an Application to the Portal Frame," Phil. Mag., vol. xvii (1934), p. 1004.

[3] "The Stability of Tall Building Frames," Inst. C.E. Selected Engineering Paper, No. 167.

section; that is, a girder with parallel chords, divided into panels of equal stiffness, but with web-posts that may be of different stiffnesses from the chord-sections. The results give the complete solution for this particular problem and they also determine, for any girder of this type, the "first analysis," which is a necessary stage in all design work, and particularly in the case of indeterminate structures.

In finding the stresses, the bending-moments are considered to be unaffected by the deformation due to direct stresses in the members, and they are obtained by consideration of the flexural strain-energy only. This approximation is inherent in all the methods of analysis commonly used for problems of this kind, and does not invalidate the results obtained, as the Author has shown.[1]

The problem is first solved for the case of rigid joints, and it is of interest to note that recent experiments on welded connections [2] have shown such joints to be constructed in practice. Deformation of the joints may be allowed for by including, in the expression for the total strain-energy, additional terms due to the rotation of the joints, and a solution is given for the case in which the deformation of the joint is proportional to the bending-moment transmitted, which is of special interest since the necessary modifications of the standard solutions can be written down at once.

Numerical results are worked out in full, and influence-lines plotted, for girders of six, eight and ten panels, with three different values in each case for the ratio, s, of the stiffnesses of web-posts and chord-sections.

The values for the bending-moments show maxima at the centres of intersection of the axes of members, and, although in practice these values will not be reached, since the moments cannot be transferred from one section to another exactly at the points of intersection, the values at these points determine the variation of bending-moment in each section between the joints.

In order to compare the results for different cases, it is convenient to consider the case in which there is a point of inflexion at the centre of every chord-section, namely $1/s=0$, as a first approximation, and to express the results for other cases in terms of their divergences from the values given by the first approximation. The results show that this divergence is reasonably small for the axial forces in the

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[1] "The Effect of Longitudinal Strain in the Members of an Elastic Network," *Phil. Mag.* vol. xix (1935), p. 59.

[2] *Vide* (*a*) C. R. Young and K. B. Jackson; *Canadian Journal of Research*, July and August, 1934.

(*b*) "The Relative Rigidity of Welded and Riveted Connections," *Engineering*, vol. cxxxix (1935), p. 101.

chord-members, but that an allowance may be necessary for the bending-moments.

The analysis, as indicated by the tabulated values and the diagrams, shows that if the stiffness-ratio can be kept at a reasonable value, say not less than 1, the first approximation will give a near estimate of the maximum forces and bending-moments.

In the last section of the Paper an approximate method is given for estimating the way in which the load is carried when the chords are not of the same stiffness and are not parallel.

The Girder of Uniform Section.

The girder is of the type indicated in *Fig. 1*, and is formed of elastic bars of uniform section, rigidly connected with one another or connected to rigid gussets by elastic joints. The two parallel

Fig. 1.

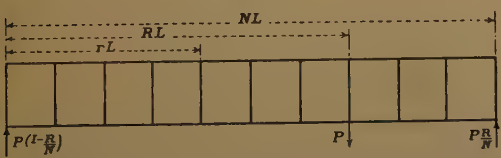


Fig. 2.

chords are each of relevant moment of inertia I, and there are $N+1$ vertical posts of moment of inertia I'; the length of each chord-member is denoted by L, and the length of each post by H. The girder is simply supported at the ends and carries a load, P, at the Rth intermediate panel-point from the left end.

Owing to the symmetry of this case, the bending-moments are the same at corresponding points in the upper and lower chords. The moments in the rth chord-section are produced by terminal couples, α_r, β_r, acting in the same direction, as indicated in *Fig. 2*. Then the terminal couple at each end of the rth post is $-(\beta_{r-1}+\alpha_r)$.

The total strain-energy due to bending can now be written down at once.[1] If w denotes the strain-energy in the rth chord-section, and w' the strain-energy in the rth post, we have

$$\left.\begin{aligned} 6EKw &= \alpha_r^2 - \alpha_r\beta_r + \beta_r^2 \\ 6EK'w' &= (\beta_{r-1}+\alpha_r)^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

where $K = I/L$, $K' = I'/H$, and E denotes Young's Modulus.

Then, if W_r is the element of the total strain-energy that contains

[1] See footnote 2 (*a*), p. 67.

α_r, β_r, that is, the strain-energy of the two chord-members in the rth panel and the two associated posts,

$$6EK'W_r = 2\frac{K'}{K}(\alpha_r^2 - \alpha_r\beta_r + \beta_r^2) + (\beta_{r-1} + \alpha_r)^2 + (\beta_r + \alpha_{r+1})^2,$$

or, if $s = K'/K$, and writing α, β for α_r, β_r

$$\begin{aligned}6EK'W_r &= \alpha^2(1 + 2s) + \beta^2(1 + 2s) - 2s\alpha\beta \\ &\quad + 2\alpha\beta_{r-1} + 2\beta\alpha_{r+1} + \beta^2{}_{r-1} + \alpha^2{}_{r+1} \quad . \quad . \quad . \quad (2)\end{aligned}$$

omitting at this stage the strain-energy due to the rotation of the joints, which is dealt with on p. 76 below.

Considering now the equilibrium of the chord-sections in the rth panel:

If $r \leq R$,

$$\begin{aligned}2(\alpha + \beta) &= P(1 - R/N)L \\ &= 2m, \text{ say} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)\end{aligned}$$

and if $r > R$,

$$\begin{aligned}2(\alpha + \beta) &= -P(R/N)L \\ &= 2cm, \text{ where } c = -R/(N - R) \quad . \quad (4)\end{aligned}$$

Then, if W is the total strain-energy of the system, Castigliano's equations of minimum strain-energy can be formed. It has been shown that α, β are not independent of one another, so that only one of the two can be taken as an independent variable. If this is α, then, from equations (3) and (4),

$$\frac{\delta\beta}{\delta\alpha} = -1,$$

and the conditions for minimum strain-energy are,

$$\frac{\delta W}{\delta\alpha_1} = \frac{\delta W}{\delta\alpha_2} = \quad . \quad . \quad . \quad . \quad . = \frac{\delta W}{\delta\alpha_N} = 0 \quad . \quad . \quad . \quad (5)$$

Now it follows from the definition of W_r that

$$\frac{\delta W}{\delta\alpha} = \frac{\delta W_r}{\delta\alpha}.$$

Thus $\quad 0 = \dfrac{\delta W_r}{\delta\alpha}$, and, from equation (2),

$$0 = 2\alpha(1 + 2s) - 2\beta(1 + 2s) - 2s(\beta - \alpha) + 2\beta_{r-1} - 2\alpha_{r-1}.$$

When $r \nless R$, this may be written

$$0 = 2\alpha(1 + 3s) - m.3s - \alpha_{r-1} - \alpha_{r-1}$$

Again, if $\qquad \sigma = -2(1 + 3s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$

$$0 = -(\sigma + 2)\frac{m}{2} + \alpha_{r-1} + \sigma\alpha + \alpha_{r+1}. \quad \ldots \ldots \quad (7)$$

When $r = 1$, the form of this equation is altered, since there is then no term β_{r-1} in equation (2), and the equation associated with the first panel becomes

$$0 = -\sigma\frac{m}{2} + \sigma\alpha_1 + \alpha_2 \quad \ldots \ldots \quad (8)$$

When $r > R + 1$, equation (7) becomes

$$0 = -c(\sigma + 2)\frac{m}{2} + \alpha_{r-1} + \sigma\alpha_r + \alpha_{r+1} \quad \ldots \quad (9)$$

Two other special cases occur, namely, when $r = R + 1$ and when $r = N$, and the two corresponding equations are

$$0 = -(2 + c\sigma)\frac{m}{2} + \alpha_R + \sigma\alpha_{R+1} + \alpha_{R+2} \quad \ldots \ldots \quad (10)$$

and

$$0 = -c(\sigma + 2)\frac{m}{2} + \alpha_{N-1} + \sigma\alpha_N \quad \ldots \ldots \quad (11)$$

Hence the following N equations are obtained to determine the N unknown moments :—

$$\left.\begin{array}{lll}
0 = & -\sigma\frac{m}{2} & +\sigma\alpha_1+\alpha_2 \\
0 = & -(\sigma+2)\frac{m}{2} & +\alpha_1+\sigma\alpha_2+\alpha_3 \\
0 = & -(\sigma+2)\frac{m}{2} & +\alpha_2+\sigma\alpha_3+\alpha_4 \\
\ldots & \ldots & \ldots \\
0 = & -(\sigma+2)\frac{m}{2} & +\alpha_{R-1}+\sigma\alpha_R+\alpha_{R+1} \\
0 = & -(2+c\sigma)\frac{m}{2} & +\alpha_R+\sigma\alpha_{R+1}+\alpha_{R+2} \\
0 = & -c(\sigma+2)\frac{m}{2} & +\alpha_{R+1}+\sigma\alpha_{R+2}+\alpha_{R+3} \\
\ldots & \ldots & \ldots \\
0 = & -c(\sigma+2)\frac{m}{2} & +\alpha_{N-2}+\sigma\alpha_{N-1}+\alpha_N \\
0 = & -c(\sigma+2)\frac{m}{2} & +\alpha_{N-1}+\sigma\alpha_N
\end{array}\right\} (12)$$

The solution is established when the values of α_1, α_R, α_{R+1}, α_N are determined, since all the moments can then be evaluated; thus

$$-\frac{m/2}{D_N} = -\frac{\alpha_1}{{}_NA_R} = (-1)^R\frac{\alpha_R}{{}_NR_R} = (-1)^{R+1}\frac{\alpha^{R+1}}{{}_NS_R} = (-1)^N\frac{\alpha_N}{{}_NN_R} \quad (13)$$

where

$$D_N = \begin{vmatrix} \sigma, & 1 & 0 & - & - & - & 0 \\ 1 & \sigma, & 1 & - & - & - & 0 \\ 0 & 1 & \sigma, & 1 & - & - & 0 \\ - & - & - & - & - & - & - \\ 0 & - & - & 1 & \sigma, & 1 & 0 \\ 0 & - & - & - & 1 & \sigma, & 1 \\ 0 & - & - & - & 0 & 1 & \sigma, \end{vmatrix}_N \quad \ldots\ldots \quad (14)$$

etc.

The following relations are then apparent:—

$$D_N = \sigma D_{N-1} - D_{N-2} \quad \ldots\ldots \quad (15)$$

$$D_N = D_R D_{N-R} - D_{R-1} D_{N-R-1} \quad \ldots \quad (16)$$

and, following from equation (15),

$$D_N = \sigma^N - (N-1)\sigma^{N-2} + \frac{(N-2)(N-3)}{1\,.\,2}\sigma^{N-4}$$

$$-\frac{(N-3)(N-4)(N-5)}{1\,.\,2\,.\,3}\sigma^{N-6} + \text{etc.} \quad .\ . \quad (17)$$

Since the value of σ, which is equal to $-2(1+3s)$, is always numerically greater than 2, it is convenient to re-write equation (17) in the form,

$$(D)_N = \frac{D_N}{\sigma_N} = 1 - (N-1)/\sigma^2 + \frac{(N-2)(N-3)}{1\,.\,2}\Big/\sigma^4 + \text{etc.}\,. \quad (18)$$

and this series is effectively convergent for all values of σ and N that are likely to be associated with the problem.

In the following analysis, the form D is used to establish the solutions, and (D) is suitable for the determination of numerical results. Values of (D) for a range of values of σ and for values of N from 1 to 10 are given in Table I.

TABLE I.—VALUES OF $(D)_N = D_N/\sigma^N$.

s	1/6	1/3	1	3	∞
σ	3·0	4·0	8.0	20·0	∞
$(D)_1$	1·000000	1·000000	1·000000	1·000000	1·000000
$(D)_2$	0·888889	0·937500	0·984375	0·997500	1·000000
$(D)_3$	0·777778	0·875000	0·968750	0·975000	1·000000
$(D)_4$	0·679012	0·816406	0·953369	0·992506	1·000000
$(D)_5$	0·592593	0·761719	0·938232	0·990019	1·000000
$(D)_6$	0·517146	0·710694	0·923336	0·987536	1·000000
$(D)_7$	0·451303	0·663086	0·908676	0·985063	1·000000
$(D)_8$	0·393842	0·618668	0·894249	0·982594	1·000000
$(D)_9$	0·343697	0·577224	0·880051	0·980131	1·000000
$(D)_{10}$	0·299937	0·538558	0·866078	0·977675	1·000000

Now, $${}_N R_R = \begin{array}{r} \\ \\ \\ \\ \\ R \\ \\ \\ \\ \\ N \end{array}\begin{vmatrix} \sigma, & \sigma & 1 & 0 & - & - & - & - & - & - & 0 \\ \sigma+2 & 1,\sigma & 1 & - & - & - & - & - & - & & 0 \\ \sigma+2 & 0 & 1,\sigma & - & - & - & - & - & - & & 0 \\ - & - & - & - & - & - & - & - & - & - & - \\ \sigma+2 & - & - & 0 & 1,\sigma & 0 & - & - & - & & 0 \\ \sigma+2 & - & - & - & 0 & 1,1 & 0 & - & - & & 0 \\ 2+c\sigma & - & - & - & - & 0 & \sigma,1 & - & - & & 0 \\ c.\sigma+2 & - & - & - & - & 0 & 1 & \sigma,1 & - & & 0 \\ - & - & - & - & - & - & - & - & - & - & - \\ c.\sigma+2 & - & - & - & - & - & - & 1 & \sigma, & & 1 \\ c.\sigma+2 & - & - & - & - & - & - & 0 & 1 & & \sigma, \end{vmatrix} \quad . \quad (19)$$

$$= \begin{array}{r} \\ \\ \\ \\ \\ R \end{array}\begin{vmatrix} \sigma, & \sigma\ 1 & - & - & - \\ \sigma+2 & 1,\sigma & 1 & - & - \\ \sigma+2 & 0 & 1,\sigma & - & - \\ - & - & - & - & - \\ \sigma+2 & - & - & 0 & 1,\sigma \\ \sigma+2 & - & - & - & 0\ 1, \end{vmatrix} . D_{N-R} + \begin{array}{r} \\ \\ \\ \\ \\ N-R \end{array}\begin{vmatrix} 2+c\sigma, & 1 & 0 & - & - & 0 \\ c.\sigma+2 & \sigma,1 & - & - & & 0 \\ c.\sigma+2 & 1 & \sigma,1 & - & & 0 \\ - & - & - & - & - & - \\ c.\sigma+2 & - & - & 1 & & \sigma,1 \\ c.\sigma+2 & - & - & - & 1 & \sigma, \end{vmatrix} . (-1)^R D_{R-1}$$

$$= \begin{array}{r} \\ \\ \\ \\ \\ R \end{array}\begin{vmatrix} -1,\sigma & 1 & - & - & 0 \\ 0 & 1,\sigma & 1 & - & 0 \\ 0 & 0 & 1,\sigma & - & 0 \\ - & - & - & - & - \\ 1 & 0 & - & - & 1,\sigma \\ \sigma+1 & 0 & - & - & 0\ 1, \end{vmatrix} . D_{N-R} + \begin{array}{r} \\ \\ \\ \\ \\ N-R \end{array}\begin{vmatrix} \sigma, & 1 & 0 & - & - & 0 \\ \sigma+2 & \sigma,1 & 0 & - & & 0 \\ \sigma+2 & 1 & \sigma,1 & - & & 0 \\ - & - & - & - & - & - \\ \sigma+2 & - & - & 1 & & \sigma,1 \\ \sigma+2 & - & - & - & 1 & \sigma, \end{vmatrix} . c(-1)^R D_{R-1}$$

$$+ 2(-1)^R D_{R-1} . D_{N-R-1}$$

$$= D_{N-R}\{\overline{\sigma + 1}D_{R-1} - D_{R-2} + (-1^{R}\}(-1)^{R-1} + 2(-1)^{R}D_{R-1} \cdot D_{N-R-1}$$

$$+ \left|\begin{array}{ccccccc} \overline{\sigma-1}, & 1 & 0 & - & - & 0 \\ 1 & \sigma, & 1 & - & - & 0 \\ 0 & 1 & \sigma, & 1 & - & 0 \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ 0 & - & - & 1 & \sigma, & 1 \\ 1 & - & - & - & 1 & \sigma, \end{array}\right|_{N-R} \cdot c(-1)^{R}D_{R-1}$$

$$= D_{N-R}\{D_R + D_{R-1} + (-1)^{R}\} \cdot (-1)^{R-1} + 2(-1)^{R}D_{R-1} \cdot D_{N-R-1} + c \cdot (-1)^{R}D_{R-1}\{D_{N-R} - D_{N-R-1} - (-1)^{N-R}\}$$

or, $${}_{N}R_{R}(-1)^{R-1} = D_{N-R}\{D_R + D_{R-1} + (-1)^{R}\} - 2D_{R-1} \cdot D_{N-R-1} + \frac{R}{N-R} \cdot D_{R-1}\{D_{N-R} - D_{N-R-1} - (-1)^{N-R}\} \quad . \quad . \quad (20)$$

Similarly, it may be shown that

$${}_{N}S_{R} \cdot (-1)^{R} = D_{N-R-1}\{D_R + D_{R-1} - (-1)^{R}\} - 2D^{R-1} \cdot D_{N-R-1} - \frac{R}{N-R} \cdot D_{R}\{D_{N-R} - D_{N-R-1} - (-1)^{N-R}\} \quad . \quad . \quad (21)$$

Thus, referring to equation (13), α_r, α_{r+1} are determined in terms of D as defined on p. 72.

When $N = 2R$, that is, when the load is at the centre of the girder,

$$(-1)^{R-1}{}_{N}R_{R} = D_R D_R + 2D_R D_{R-1} - 3D_{R-1}D_{R-1} + (-1)^{R}(D_R - D_{R-1})$$

and $$\alpha_R = (-1)^{R-1}\frac{{}_{N}R_{R}}{D_N}\frac{m}{2},$$

$$= \frac{(D_R - D_{R-1})(D_R + 3D_{R-1} + (-1)^{R})}{D_R D_R - D_{R-1}D_{R-1}}\frac{m}{2}$$

$$= \frac{D_R + 3D_{R-1} + (-1)^{R}}{D_R + D_{R-1}}\frac{PL}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and similarly,

$$\alpha_{R+1} = -\frac{D_R - D_{R-1} - (-1)^{R}}{D_R + D_{R-1}}\frac{PL}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

From these two results, and using the relation expressed in equation (3), it follows that, in this case, $\beta_r + \alpha_{r+1} = 0$, and there is no bending-moment in the centre post, a conclusion that is to be expected from the symmetry of this particular case. It is interesting to note that, here, either half of the girder corresponds to a cantilever of R panels loaded at one end, and the case is identical with that of a tall building frame of similar form with a horizontal load at the top.

Equations (20) and (21) might be said to determine the solution of equations (12), but in practice it is possible to solve by substitution only for a short sequence of unknowns, and so, for convenience in obtaining numerical results, it is desirable to add the solutions for α_1 and α_N.

$$
\text{Thus,}\quad {}_N A_R = \begin{array}{r} \\ \\ \\ \\ R \\ \\ \\ \\ \\ N \end{array}
\left|\begin{array}{cccccccccc}
\sigma, & 1 & 0 & - & - & - & - & - & - & 0 \\
\overline{\sigma+2} & \sigma, & 1 & - & - & - & - & - & - & 0 \\
\overline{\sigma+2} & 1 & \sigma, & 1 & - & - & - & - & - & 0 \\
- & - & - & - & - & - & - & - & - & - \\
\overline{\sigma+2} & - & - & 1 & \sigma, & 1 & - & - & - & 0 \\
\overline{2+c\sigma} & - & - & - & 1 & \sigma, & 1 & - & - & 0 \\
c.\overline{\sigma+2} & - & - & - & - & 1 & \sigma, & 1 & - & 0 \\
- & - & - & - & - & - & - & - & - & - \\
c.\overline{\sigma+2} & - & - & - & - & - & - & 1 & \sigma, & 1 \\
c.\overline{\sigma+2} & - & - & - & - & - & - & - & 1 & \sigma,
\end{array}\right|
$$

$$
\begin{aligned}
&= \sigma D_{N-1} - {}_{N-1}A_{R-1} - 2D_{N-2} \\
&= D_N - D_{N-2} - {}_{N-1}A_{R-1} \\
&= (D_N - D_{N-1}) + (D_{N-1} - D_{N-2}) - {}_{N-1}A_{R-1} \\
&= (D_N - D_{N-1}) - (-1)^R(D_{N-R} - D_{N-R-1} - {}_{N-R}A_{R-R}) \\
&= (D_N - D_{N-1}) - (-1)^R(D_{N-R} - D_{N-R-1})
\end{aligned}
$$

$$
+(-1)^R . \begin{array}{r} \\ \\ \\ \\ \\ N-R \end{array}
\left|\begin{array}{cccccc}
\underline{c\sigma,} & 1 & 0 & - & - & 0 \\
c.\overline{\sigma+2} & \sigma, & 1 & 0 & - & 0 \\
c.\overline{\sigma+2} & 1 & \sigma, & 1 & - & 0 \\
- & - & - & - & - & - \\
c.\overline{\sigma+2} & - & - & 1 & \sigma, & 1 \\
c.\overline{\sigma+2} & - & - & - & 1 & \sigma,
\end{array}\right|
$$

$$
\begin{aligned}
&= (D_N - D_{N-1}) - (-1)^R(D_{N-R} - D_{N-R-1}) \\
&\qquad + (-1)^R . c. \{D_{N-R} - D_{N-R-1} - (-1)_{N-R}\},
\end{aligned}
$$

whence

$$ {}_N A_R = (D_N - D_{N-1}) - (-1)^R (D_{N-R} - D_{N-R-1}) \frac{N}{N-R} $$
$$ + (-1)^N \frac{R}{N-R} \quad . \quad . \quad (24) $$

Similarly, it may be shown that

$$ (-1)^{N-1} {}_N N_R = -\frac{R}{N-R} (D_N + D_{N-1}) $$
$$ - (-1)^{N-R} (D_R - D_{R-1}) \frac{N}{N-R} + (-1)^N \quad . \quad . \quad (25) $$

When $N = 2R$,

$$ {}_N A_R = D_{2R} - D_{2R-1} - 2(-1)^R (D_R - D_{R-1}) + 1 ; $$

hence, $$ \sigma_1 = \frac{D_{2R} - D_{2R-1} - 2(-1)^R (D_R - D_{R-1}) + 1}{D_{2R}} \frac{m}{2} $$

$$ = \frac{D_R D_R - D_{R-1} D_{R-1} - D_R D_{R-1} + D_{R-1} D_{R-2} - 2(-1)^R (D_R - D_{R-1}) + 1}{D_R D_R - D_{R-1} D_{R-1}} \frac{m}{2} $$

$$ = \frac{(D_R - D_{R-1})(D_R - D_{R-2}) - D_{R-1} D_{R-1} + D_R D_{R-2} - 2(-1)^R (D_R - D_{R-1}) + 1}{(D_R + D_{R-1})(D_R - D_{R-1})} \frac{m}{2} $$

Now, $$ D_{R-1} D_{R-1} - D_R D_{R-2} = D_{R-2} D_{R-2} - D_{R-1} D_{R-3} $$
$$ = \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot $$
$$ = D_1 D_1 - D_2 $$
$$ = 1 , $$

and also $$ \alpha_1 = \frac{D_R - D_{R-2} - 2(-1)^R}{D_R + D_{R-1}} \frac{PL}{8} \quad . \quad . \quad . \quad . \quad (26) $$

Similarly, $$ \alpha_N = -\frac{D_R + 2D_{R-1} + D_{R-2} + 2(-1)^R}{D_R + D_{R-1}} \frac{PL}{8} \quad (27) $$

when $N = 2R$.

These two results are comparable with equations (22) and (23), and may be similarly related to the case in which a similar structure is supported and loaded as a cantilever.

If the joints connecting the members of the framework with one another are not rigid, the strain-energy of the joints may be included in the expression for the total strain-energy, given in equation (2). The strain-energy of a joint will be a function of the moment transmitted by it, and the modified framework can still be solved by the application of equations (5). The relation between the moment applied to a joint and the angular deformation produced may be determined experimentally; a discussion of experimental results is outside the scope of this Paper, but it may be noted that recent investigations do not preclude an approximately linear relation between moment and deformation for the type of heavy connection required in frame-girder work. If the relation is linear, the strain-energy w will be proportional to the square of the bending-moment, and may be written in the form

$$6EK'w = km^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

If now the coefficient for the joint at the end of a chord-member is k_1, and the coefficient for the joint at the end of a post is k_2, the strain-energy in the rth panel becomes

$$\begin{aligned} 6EK'W_r &= \alpha^2(1+2s) + \beta^2(1+2s) - 2s\alpha\beta + 2\alpha\beta_{r-1} + 2\beta\alpha_{r+1} \\ &\quad + \beta^2{}_{r-1} + \alpha^2{}_{r+1} + 2k_1\alpha^2 + 2k_1\beta^2 + 2k_2(\alpha+\beta_{r-1})^2 \\ &\quad + 2k^2(\beta+\alpha_{r+1})^2 \\ &= (1+2s+2k_1+2k_2)(\alpha^2+\beta^2) - 2s\alpha\beta + 2(1+2k_2)\alpha\beta_{r-1} \\ &\quad + 2(1+2k_2)\beta\alpha_{r+1} + \beta^2{}_{r-1}(1+2k_2) + \alpha^2{}_{r+1}(1+2k_2) \quad . \quad (29) \end{aligned}$$

Then, as on p. 70, $\qquad 0 = \dfrac{\delta W_r}{\delta\alpha}$;

hence,

$$\begin{aligned} 0 = 2(\alpha-\beta)(1+2s+2k_1+2k_2) - 2s(\beta-\alpha) \\ + 2(1+2k_2)(\beta_{r-1} - \alpha_{r+1}); \end{aligned}$$

and since $\qquad \alpha + \beta = \dfrac{m}{2}$,

$$0 = 2\alpha\left(1 + \frac{3s+2k_1}{1+2k_2}\right) - m\left(\frac{3s+2k_1}{1+2k_2}\right) - (\alpha_{r-1} + \alpha_{r+1}).$$

Then if $\qquad \sigma = -2\left(1 + \dfrac{3s+2k_1}{1+2k_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$

$$0 = -(\sigma+2)\frac{m}{2} + \alpha_{r-1} + \sigma\alpha + \alpha_{r+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

which is identical with equation (7) and the former analysis is applicable.

The axial forces in the chords may be found by taking a section (*Fig. 3*) through the points of inflexion in the chord-member for which the force F is required.

If α, $-\beta$, are the moments at the ends of a chord-member, the bending-moment at any point distant xL from the right-hand end of the member is

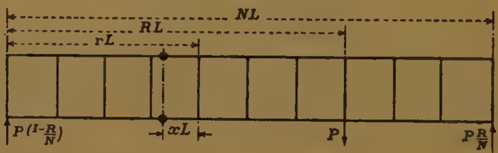
$$M = \alpha x - \beta(1 - x)$$

and the point of inflexion is given by $M = 0$,

or,
$$x = \beta/(\alpha + \beta), \quad \ldots \ldots \quad (32)$$

where x may, of course, be greater than 1.

Fig. 3.



When the load is on the right of the section, that is, if $R \geqslant r$,

$$FH = P\frac{(N-R)}{N}(rL - xL)$$

$$= PL\frac{(N-R)}{N}(r-1) + 2\alpha,$$

or,
$$F = \frac{PL}{H}\frac{(N-R)}{N}(r-1) + \frac{2\alpha}{H},$$

where H is the depth of the girder.

When this load is on the left of the section, that is, if $R < r$,

$$FH = P\frac{N}{R}(N - r + x)L$$

$$= PL\frac{R}{N}(N-r) + 2\beta,$$

or,
$$F = \frac{PL}{H}\frac{R}{N}(N-r) + \frac{2\beta}{H}.$$

Numerical values have been worked out for three different values of N, and for three different values of the stiffness-ratio, and these values are given in the following Tables II–X. These Tables are illustrated by diagrams (*Figs. 4* to *6*), which show

TABLE II.

$N = 6;\ s = 1.$

Values of the chord bending-moments.[1]

	$R=1$	$R=2$	$R=3$	$R=4$	$R=5$	Sum.	Maximum	Minimum
α_1	1·5921	1·4663	1·1224	0·7507	0·3756	5·3071	5·3071	0
β_1	1·7412	1·2003	0·8776	0·5826	0·2911	4·6928	4·6928	0
α_2	—0·5968	1·0639	0·9792	0·6728	0·3382	2·4573	2·8629	—0·4056
β_2	—0·0698	1·6027	1·0208	0·6605	0·3285	3·5427	3·5791	—0·0364
α_3	—0·3667	—0·9547	0·7113	0·6314	0·3298	0·3511	1·4687	—1·1176
β_3	—0·3000	—0·3786	1·2887	0·7019	0·3368	1·6488	2·1811	—0·5323
Values of the post bending-moments.[1]								
α_1	1·5921	1·4663	1·1224	0·7507	0·3756	5·3071	5·3701	0
$\beta_1 + \alpha_2$	1·1444	2·2643	1·8568	1·2554	0·6293	7·1501	7·1501	0
$\beta_2 + \alpha_3$	—0·4365	0·6480	1·7321	1·2919	0·6583	3·8938	4·1999	—0·3061
$\beta_3 + \alpha_4$	—0·6368	—1·0805	0	1·0805	0·6368	0	1·7173	—1·7173
Values of the axial forces in the chords.[2]								
Panel Number 1	0·3980	0·3666	0·2806	0·1877	0·0939	1·3268	—	—
2	0·6492	0·9327	0·7448	0·5015	0·2512	3·0676	—	—
3	0·5750	1·0947	1·1778	0·8245	0·4158	4·0878	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE III.

$N = 6$; $s = \frac{1}{3}$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	Sum.	Maximum	Minimum
α_1	1·4338	1·5088	1·2195	0·8327	0·4198	5·4146	5·4146	0
β_1	1·8995	1·1579	0·7805	0·5006	0·2469	4·5854	4·5854	0
α_2	—0·9312	0·7017	0·8780	0·6642	0·3458	1·6585	2·3896	—0·7311
β_2	0·2645	1·9650	1·1220	0·6692	0·3208	4·3415	4·3415	0
α_3	—0·4919	—1·3686	0·2927	0·4906	0·2968	—0·7804	—1·7399	0·9595
β_3	—0·1748	0·0353	1·7073	0·8428	0·3698	2·7804	2·9405	—0·1601
Values of the post bending-moments.[1]								
α_1	1·4338	1·5088	1·2195	0·8327	0·4198	5·4146	5·4146	0
$\beta_1 + \alpha_2$	0·9683	1·8596	1·6585	1·1648	0·5927	6·2439	6·2439	0
$\beta_2 + \alpha_3$	—0·2274	0·5964	1·4146	1·1597	0·6177	3·5610	3·7061	—0·1451
$\beta_3 + \alpha_4$	—0·5446	—0·8075	0	0·8075	0·5446	0	1·3521	—1·3521
Values of the axial forces in the chords.[2]								
Panel Number 1	0·3585	0·3772	0·3049	0·2082	0·1049	1·3537	—	—
2	0·6005	0·8421	0·7195	0·4994	0·2531	2·9146	—	—
3	0·5437	0·9912	1·0732	0·7893	0·4075	3·8049	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE IV.

$N = 6;\ 1/s = 0.$

Values of the chord bending-moments.[1]

	$R=1$	$R=2$	$R=3$	$R=4$	$R=5$	Sum.	Maximum	Minimum
α_1	1·6667	1·3333	1·0000	0·6667	0·3333	5·0000	5·0000	0
β_1	1·6667	1·3333	1·0000	0·6667	0·3333	5·0000	5·0000	0
α_2	−0·3333	1·3333	1·0000	0·6667	0·3333	3·0000	3·2000	−0·2000
β_2	−0·3333	1·3333	1·0000	0·6667	0·3333	3·0000	3·2000	−0·2000
α_3	−0·3333	−0·6667	1·0000	0·6667	0·3333	1·0000	1·8000	−0·8000
β_3	−0·3333	−0·6667	1·0000	0·6667	0·3333	1·0000	1·8000	−0·8000

Values of the post bending-moments.[1]

	$R=1$	$R=2$	$R=3$	$R=4$	$R=5$	Sum.	Maximum	Minimum
α_1	1·6667	1·3333	1·0000	0·6667	0·3333	5·0000	5·0000	0
$\beta_1 + \alpha_2$	1·3333	2·6666	2·0000	1·3333	0·6667	8·0000	8·0000	0
$\beta_2 + \alpha_3$	−0·6667	0·6667	2·0000	1·3333	0·6667	4·0000	4·5000	−0·5000
$\beta_3 + \alpha_4$	−0·6667	−1·3333	0	1·3333	0·6667	0	2·0000	−2·0000

Values of the axial forces in the chords.[2]

	$R=1$	$R=2$	$R=3$	$R=4$	$R=5$	Sum.	Maximum	Minimum
Panel Number 1	0·4167	0·3333	0·2500	0·1667	0·0833	1·2500	—	—
2	0·7500	1·0000	0·7500	0·5000	0·2500	3·2500	—	—
3	0·5833	1·1667	1·2500	0·8333	0·4167	4·2500	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE V.

$N = 8$; $s = 1$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	Sum.	Maximum	Minimum
α_1	1·6860	1·6542	1·4042	1·1264	0·8452	0·5635	0·2818	7·5613	7·5613	0
β_1	1·8140	1·3458	1·0998	0·8736	0·6548	0·4365	0·2182	6·4387	6·4387	0
α_2	—0·5122	1·2333	1·2332	1·0114	0·7615	0·5080	0·2540	4·4892	4·8204	—0·3312
β_2	0·0122	1·7667	1·2668	0·9886	0·7385	0·4920	0·2460	5·5108	5·5108	0
α_3	—0·2833	—0·7879	0·9616	0·9651	0·7468	0·5009	0·2506	2·3538	3·2084	—0·8546
β_3	—0·2167	—0·2121	1·5384	1·0349	0·7532	0·4991	0·2494	3·6462	3·9818	—0·3356
α_4	—0·2542	—0·5366	—1·0406	0·7093	0·7133	0·4951	0·2500	0·3363	1·9568	—1·6205
β_4	—0·2458	—0·4634	—0·4594	1·2907	0·7867	0·5049	0·2500	1·6637	2·5529	—0·9992
Values of the post bending-moments.[1]										
α_1	1·6860	1·6542	1·4042	1·1264	0·8452	0·5635	0·2818	7·5613	7·5613	0
$\beta_1 + \alpha_2$	1·3018	2·5791	2·3290	1·8850	1·4163	0·9445	0·4722	10·9279	10·9279	0
$\beta_2 + \alpha_3$	—0·2711	0·9788	2·2284	1·9537	1·4853	0·9929	0·4966	7·8646	8·0295	—0·1649
$\beta_3 + \alpha_4$	—0·4709	—0·7487	0·4978	1·7442	1·4665	0·9942	0·4994	3·9825	5·0526	—1·0701
$\beta_4 + \alpha_5$	—0·4958	—0·9683	—1·2461	0	1·2461	0·9683	0·4958	0	2·7102	—2·7102
Values of the axial forces in the chords.[2]										
Panel Number 1	0·4215	0·4135	0·3510	0·2816	0·2113	0·1409	0·0705	1·8903	—	—
2	0·7469	1·0583	0·9333	0·7529	0·5654	0·3770	0·1885	4·6223	—	—
3	0·6792	1·3030	1·4904	1·2413	0·9367	0·6252	0·3126	6·5885	—	—
4	0·5614	1·1159	1·6149	1·6773	1·3033	0·8738	0·4375	7·5841	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE VI.

$N = 8$; $s = \frac{1}{3}$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	Sum.	Maximum	Minimum
α_1	1·5394	1·7199	1·5362	1·2549	0·9475	0·6332	0·3168	7·9479	9·9479	0
β_1	1·9606	1·2801	0·9638	0·7451	0·5525	0·3668	0·1832	6·0521	6·0521	0
α_2	—0·8423	0·8795	1·1447	1·0196	0·7900	0·5323	0·2671	3·7909	4·4181	—0·6272
β_2	0·3423	2·1205	1·3553	0·9804	0·7100	0·4677	0·2329	6·2091	6·2091	0
α_3	—0·4086	—1·2020	0·5425	0·8235	0·7124	0·4962	0·2517	1·2157	2·6394	—1·4237
β_3	—0·0914	0·2020	1·9575	1·1765	0·7876	0·5038	0·2483	4·7843	4·8442	—0·0599
α_4	—0·2922	—0·6875	—1·4745	0·2745	0·5595	0·4522	0·2398	—0·9282	—2·3385	1·4103
β_4	—0·2078	—0·3125	—0·0255	1·7255	0·9405	0·5478	0·2602	2·9282	3·4614	—0·5332

Values of the post bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	Sum.	Maximum	Minimum
α_1	1·5394	1·7109	1·5362	1·2549	0·9475	0·6332	0·3168	7·9479	7·9479	0
$\beta_1 + \alpha_2$	1·1183	2·1596	2·1085	1·7647	1·3425	0·8991	0·4503	9·8430	9·8430	0
$\beta_2 + \alpha_3$	—0·0663	0·9185	1·8978	1·8039	1·4224	0·9639	0·4846	7·4248	7·4602	—0·0354
$\beta_3 + \alpha_4$	—0·3836	—0·4855	0·4830	1·4510	1·3471	0·9560	0·4881	3·8561	4·6041	—0·7480
$\beta_4 + \alpha_5$	—0·4680	—0·8603	—0·9660	0	0·9660	0·8603	0·4680	0	2·2943	—2·2943

Values of the axial forces in the chords.[2]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	Sum.	Maximum	Minimum
Panel Number 1	0·3849	0·4300	0·3840	0·3137	0·2369	0·1583	0·0792	1·9870	—	—
2	0·6644	0·9699	0·9112	0·7549	0·5725	0·3831	0·1913	4·4472	—	—
3	0·6478	1·1995	1·3856	1·2059	0·9281	0·6241	0·3129	6·3039	—	—
4	0·5519	1·0781	1·5064	1·5686	1·2649	0·8631	0·4349	7·2639	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE VII.

$N = 8$; $1/s = 0$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	Sum.	Maximum	Minimum
α_1	1·7500	1·5000	1·2500	1·0000	0·7500	0·5000	0·2500	7·0000	7·0000	0
β_1	1·7500	1·5000	1·2500	1·0000	0·7500	0·5000	0·2500	7·0000	7·0000	0
α_2	−0·2500	1·5000	1·2500	1·0000	0·7500	0·5000	0·2500	5·0000	5·1429	−0·1429
β_2	−0·2500	1·5000	1·2500	1·0000	0·7500	0·5000	0·2500	5·0000	5·1429	−0·1429
α_3	−0·2500	−0·5000	1·2500	1·0000	0·7500	0·5000	0·2500	3·0000	3·5714	−0·5714
β_3	−0·2500	−0·5000	1·2500	1·0000	0·7500	0·5000	0·2500	3·0000	3·5714	−0·5714
α_4	−0·2500	−0·5000	−0·7500	1·0000	0·7500	0·5000	0·2500	1·0000	2·2857	−1·2857
β_4	−0·2500	−0·5000	−0·7500	1·0000	0·7500	0·5000	0·2500	1·0000	2·2857	−1·2857
Values of the post bending-moments.[1]										
α_1	1·7500	1·5000	1·2500	1·0000	0·7500	0·5000	0·2500	7·0000	7·0000	0
$\beta_1 + \alpha_2$	1·5000	3·0000	2·5000	2·0000	1·5000	1·0000	0·5000	12·0000	12·0000	0
$\beta_2 + \alpha_3$	−0·5000	1·0000	2·5000	2·0000	1·5000	1·0000	0·5000	8·0000	8·3333	−0·3333
$\beta_3 + \alpha_4$	−0·5000	−1·0000	0·5000	2·0000	1·5000	1·0000	0·5000	4·0000	5·3333	−1·3333
$\beta_4 + \alpha_5$	−0·5000	−1·0000	−1·5000	0	1·5000	1·0000	0·5000	0	3·0000	−3·0000
Values of the axial forces in the chords.[2]										
Panel Number 1	0·4375	0·3750	0·3125	0·2500	0·1875	0·1250	0·0625	1·7500	—	—
2	0·8125	1·1125	0·9375	0·7500	0·5625	0·3750	0·1875	4·7500	—	—
3	0·6875	1·3750	1·5625	1·2500	0·9375	0·6250	0·3125	6·7500	—	—
4	0·5625	1·1250	1·6875	1·7500	1·3125	0·8750	0·4375	7·7500	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE VIII.

$N = 10$; $s = 1$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
α_1	1·7423	1·7669	1·5732	1·3518	1·1269	0·9016	0·6762	0·4508	0·2254	9·8151	9·8151	0
β_1	1·8577	1·4331	1·2268	1·0482	0·8731	0·6984	0·5238	0·3492	0·1746	8·1848	8·1848	0
α_2	−0·4614	1·3349	1·3856	1·2147	1·0155	0·8128	0·6097	0·4065	0·2032	6·5215	6·8115	−0·2899
β_2	0·0614	1·8651	1·4144	1·1853	0·9845	0·7872	0·5903	0·3936	0·1968	7·4784	7·4784	0
α_3	−0·2332	−0·6877	1·1119	1·1655	0·9974	0·8010	0·6012	0·4008	0·2004	4·3573	5·0657	−0·7084
β_3	−0·1668	−0·1123	1·6881	1·2345	1·0026	0·7990	0·5989	0·3992	0·1996	5·6427	5·8692	−0·2265
α_4	−0·2042	−0·4365	−0·8906	0·9093	0·9633	0·7955	0·5996	0·4000	0·2000	2·3364	3·6428	−1·3064
β_4	−0·1958	−0·3635	−0·3094	1·4907	1·0367	0·8045	0·6005	0·4000	0·2000	3·6636	4·4041	−0·7405
α_5	−0·2005	−0·4046	−0·6369	−1·0910	0·7090	0·7631	0·5953	0·3994	0·2000	0·3338	2·4519	−2·1181
β_5	−0·1995	−0·3954	−0·5631	−0·5090	1·2910	0·8369	0·6047	0·4006	0·2000	1·6662	3·1506	−1·4844

Values of the post bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
α_1	1·7423	1·7669	1·5732	1·3518	1·1269	0·9016	0·6762	0·4508	0·2254	9·8151	9·8151	0
$\beta_1 + \alpha_2$	1·3963	2·7680	2·6124	2·2628	1·8886	1·5112	1·1335	0·7556	0·3778	14·7064	14·7064	0
$\beta_2 + \alpha_3$	−0·1718	1·1774	2·5262	2·3508	1·9818	1·5882	1·1915	0·7944	0·3972	11·8357	11·9326	−0·0969
$\beta_3 + \alpha_4$	−0·3710	−0·5488	0·7975	2·1438	1·9659	1·5945	1·1984	0·7992	0·3996	7·9791	8·7364	−0·7573
$\beta_4 + \alpha_5$	−0·3963	−0·7681	−0·9463	0·3997	1·7457	1·5676	1·1958	0·7994	0·3999	3·9974	5·9676	−1·9702
$\beta_5 + \alpha_6$	−0·3995	−0·7959	−1·1678	−1·3460	0	1·3460	1·1678	0·7959	0·3995	0	3·7092	−3·7092

Values of the axial forces in the chords.[2]

Panel	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
Number 1	0·4356	0·4417	0·3933	0·3379	0·2817	0·2254	0·1691	0·1127	0·0564	2·4538	—	—
2	0·7847	1·1337	1·0464	0·9037	0·7539	0·6032	0·4524	0·3016	0·1508	6·1304	—	—
3	0·7417	1·4281	1·6780	1·4914	1·2493	1·0003	0·7503	0·5002	0·2501	9·0894	—	—
4	0·6489	1·2909	1·8774	2·0273	1·7408	1·3989	1·0499	0·7000	0·3500	11·0841	—	—
5	0·5499	1·0989	1·6408	2·1272	2·1773	1·7908	1·3488	0·8998	0·4500	12·0835	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

Table IX.

$N = 10$; $s = \frac{1}{8}$.

Values of the chord bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
α_1	1·6028	1·8467	1·7263	1·5085	1·2645	1·0134	0·7605	0·5071	0·2536	10·4834	10·4834	0
β_1	1·9972	1·3533	1·0737	0·8915	0·7355	0·5866	0·4395	0·2929	0·1464	7·5166	7·5166	0
α_2	−0·7886	0·9866	1·3054	1·2339	1·0578	0·8537	0·6421	0·4285	0·2143	5·9335	6·5030	−0·5695
β_2	0·3886	2·2134	1·4946	1·1661	0·9422	0·7463	0·5579	0·3715	0·1857	8·0665	8·0665	0
α_3	−0·3578	−1·1004	0·6952	1·0270	0·9667	0·8013	0·6078	0·4067	0·2036	3·2503	4·4954	−1·2451
β_3	−0·0422	0·3004	2·1048	1·3730	1·0333	0·7987	0·5922	0·3933	0·1964	6·7497	6·7734	−0·0237
α_4	−0·2422	−0·5876	−1·3247	0·4742	0·8091	0·7517	0·5891	0·3984	0·2002	1·0683	3·0483	−1·9800
β_4	−0·1578	−0·2124	0·1247	1·9258	1·1909	0·8483	0·6109	0·4016	0·1998	4·9317	5·2626	−0·3309
α_5	−0·2112	−0·4501	−0·7940	−1·5301	0·2697	0·6054	0·5488	0·3871	0·1972	−0 9773	−2·8708	1·8935
β_5	−0·1888	−0·3499	−0·4060	−0·0699	1·7303	0·9946	0·6512	0·4129	0·2028	2·9773	3·9582	0·9809

Values of the post bending-moments.[1]

	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
α_1	1·6028	1·8467	1·7263	1·5085	1·2645	1·0134	0·7605	0·5071	0·2536	10·4834	10·4834	0
$\beta_1 + \alpha_2$	1·2085	2·3399	2·3790	2·1254	1·7933	1·4403	1·0816	0·7213	0·3607	13·4501	13·4501	0
$\beta_2 + \alpha_3$	0·0309	1·1130	2·1898	2·1931	1·9089	1·5476	1·1657	0·7782	0·3893	11·3167	11·3167	0
$\beta_3 + \alpha_4$	−0·2845	−0·2872	0·7801	1·8472	1·8424	1·5504	1·1813	0·7917	0·3966	7·8180	8·2847	−0·4667
$\beta_4 + \alpha_5$	−0·3690	−0·6625	−0·6693	0·3957	1·4606	1·4537	1·1596	0·7886	0·3970	3·9544	5·5309	−1·5765
$\beta_5 + \alpha_6$	−0·3915	−0·7628	−1·0572	−1·0645	0	1·0645	1·0572	0·7628	0·3915	0	3·2760	−3·2760

Values of the axial forces in the chords.[2]

Panel Number	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$	$R = 9$	Sum.	Maximum	Minimum
1	0·4007	0·4617	0·4316	0·3771	0·3161	0·2533	0·1901	0·1268	0·0634	2·6208	—	—
2	0·7028	1·0466	1·0264	0·9085	0·7645	0·6134	0·4605	0·3071	0·1536	5·9834	—	—
3	0·7105	1·3249	1·5738	1·4568	1·2417	1·0003	0·7519	0·5017	0·2509	8·8125	—	—
4	0·6394	1·2531	1·7688	1·9186	1·7023	1·3879	1·0473	0·6996	0·3501	10·7671	—	—
5	0·5472	1·0875	1·6105	2·0175	2·0674	1·7513	1·3372	0·8968	0·4493	11·7557	—	—

[1] All figures to be multiplied by $PL/8$.

[2] All figures to be multiplied by PL/H.

TABLE X.

$N = 10;\ 1/s = 0.$

Values of the chord bending-moments.[1]

	R = 1	R = 2	R = 3	R = 4	R = 5	R = 6	R = 7	R = 8	R = 9	Sum.	Maximum	Minimum
α_1	1·80	1·60	1·40	1·20	1·00	0·80	0·60	0·40	0·20	9·00	9·00	0
β_1	1·80	1·60	1·40	1·20	1·00	0·80	0·60	0·40	0·20	9·00	9·00	0
α_2	—0·20	1·60	1·40	1·20	1·00	0·80	0·60	0·40	0·20	7·00	7·11	—0·11
β_2	—0·20	1·60	1·40	1·20	1·00	0·80	0·60	0·40	0·20	7·00	7·11	—0·11
α_3	—0·20	—0·40	1·40	1·20	1·00	0·80	0·60	0·40	0·20	5·00	5·44	—0·44
β_3	—0·20	—0·40	1·40	1·20	1·00	0·80	0·60	0·40	0·20	5·00	5·44	—0·44
α_4	—0·20	—0·40	—0·60	1·20	1·00	0·80	0·60	0·40	0·20	3·00	4·00	—1·00
β_4	—0·20	—0·40	—0·60	1·20	1·00	0·80	0·60	0·40	0·20	3·00	4·00	—1·00
α_5	—0·20	—0·40	—0·60	—0·80	1·00	0·80	0·60	0·40	0·20	1·00	2·78	—1·78
β_5	—0·20	—0·40	—0·60	—0·80	1·00	0·80	0·60	0·40	0·20	1·00	2·78	—1·78

Values of the post bending-moments.[1]

	R = 1	R = 2	R = 3	R = 4	R = 5	R = 6	R = 7	R = 8	R = 9	Sum.	Maximum	Minimum
α_1	1·80	1·60	1·40	1·20	1·00	0·80	0·60	0·40	0·20	9·00	9·00	0
$\beta_1 + \alpha_2$	1·60	3·20	2·80	2·40	2·00	1·60	1·20	0·80	0·40	16·00	16·00	0
$\beta_2 + \alpha_3$	—0·40	1·20	2·80	2·40	2·00	1·60	1·20	0·80	0·40	12·00	12·25	—0·25
$\beta_3 + \alpha_4$	—0·40	—0·80	0·80	2·40	2·00	1·60	1·20	0·80	0·40	8·00	9·00	—1·00
$\beta_4 + \alpha_5$	—0·40	—0·80	—1·20	0·40	2·00	1·60	1·20	0·80	0·40	4·00	6·25	—2·25
$\beta_5 + \alpha_6$	—0·40	—0·80	—1·20	—1·60	0	1·60	1·20	0·80	0·40	0	4·00	—4·00

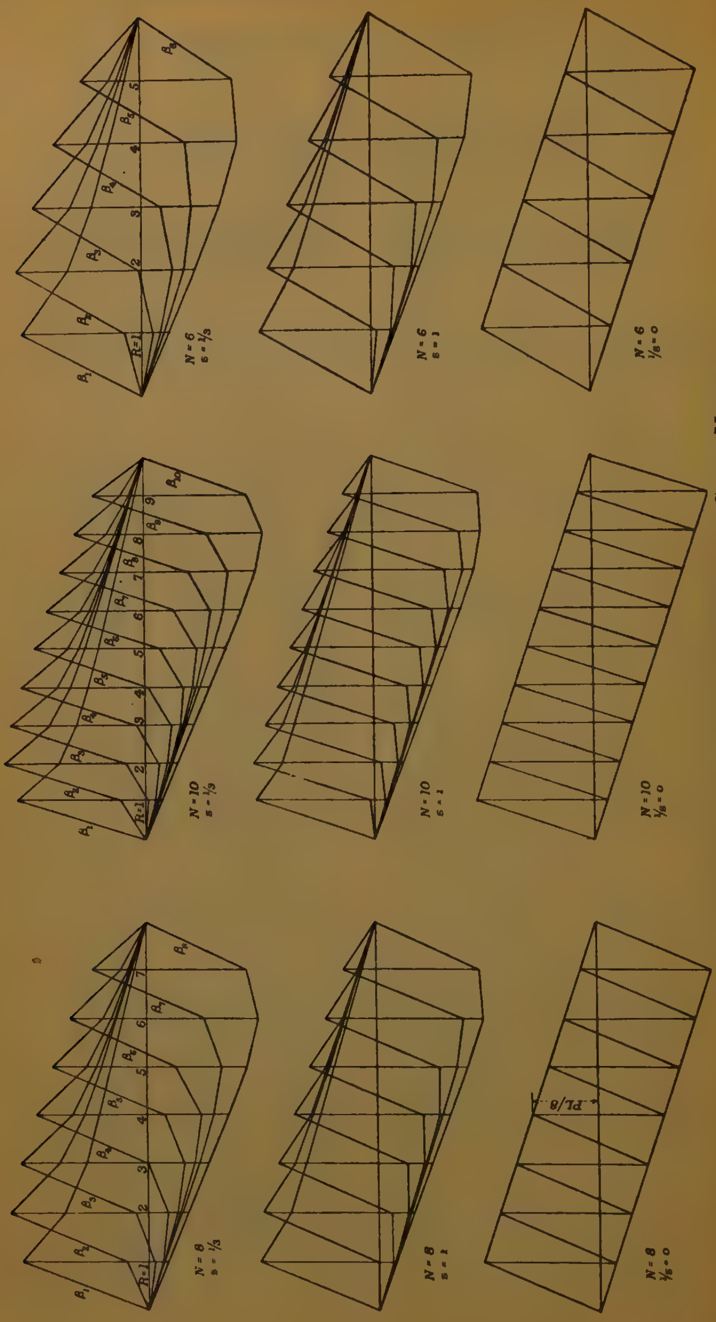
Values of the axial forces in the chords.[2]

	R = 1	R = 2	R = 3	R = 4	R = 5	R = 6	R = 7	R = 8	R = 9	Sum.	Maximum	Minimum
Panel Number 1	0·45	0·40	0·35	0·30	0·25	0·20	0·15	0·10	0·05	2·25	—	—
2	0·85	1·20	1·05	0·90	0·75	0·60	0·45	0·30	0·15	6·25	—	—
3	0·75	1·50	1·75	1·50	1·25	1·00	0·75	0·50	0·25	9·25	—	—
4	0·65	1·30	1·95	2·10	1·75	1·40	1·05	0·70	0·35	11·25	—	—
5	0·55	1·10	1·65	2·20	2·25	1·80	1·35	0·90	0·45	12·25	—	—

[1] All figures to be multiplied by $PL/8$.

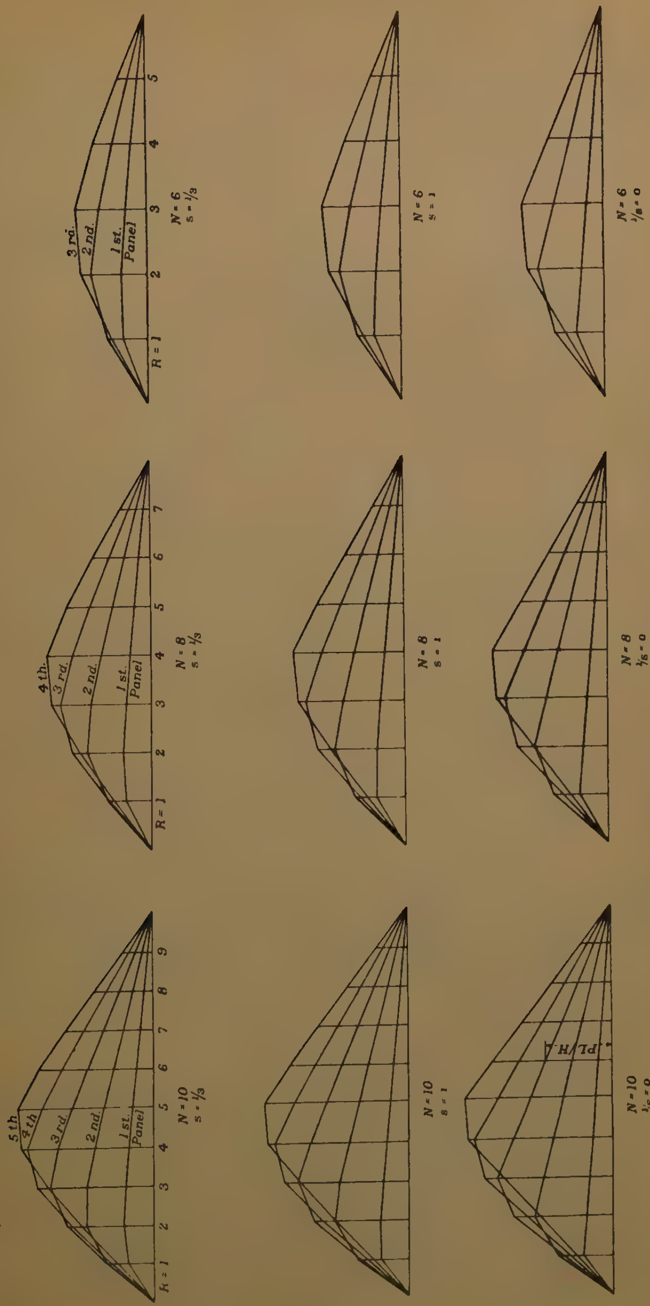
[2] All figures to be multiplied by PL/H.

Figs. 4.



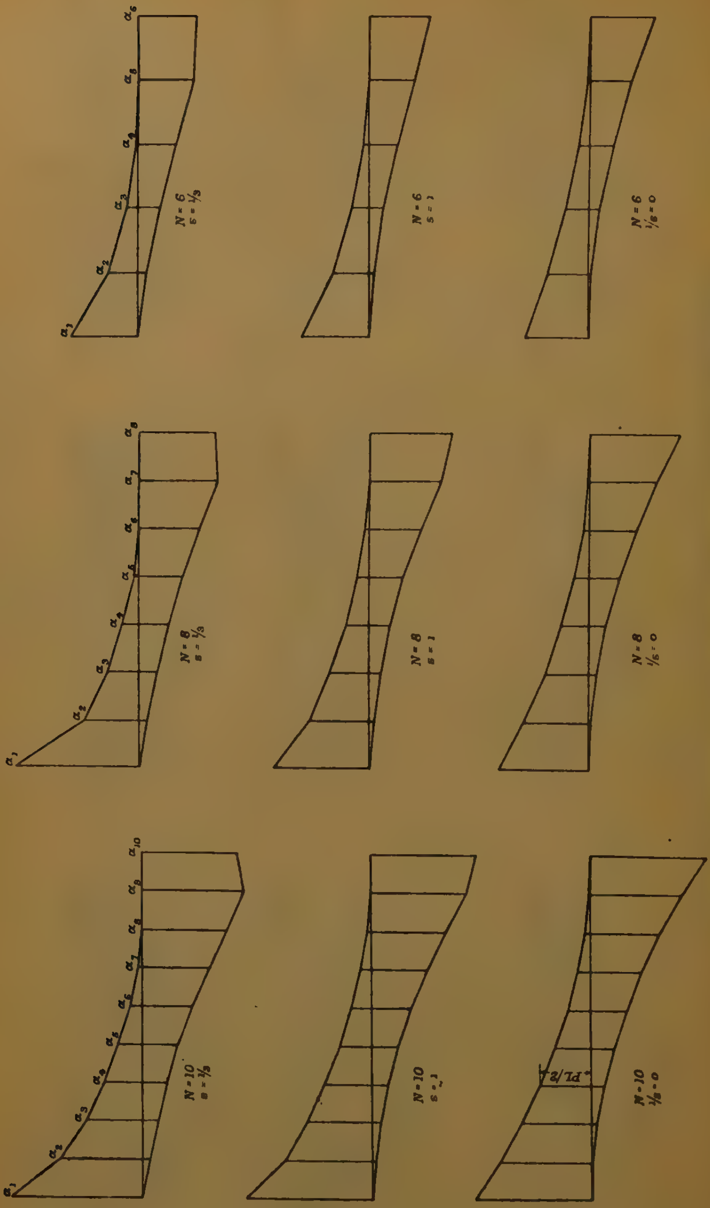
INFLUENCE-LINES FOR BENDING MOMENTS IN CHORD-MEMBERS.

Figs. 5.



INFLUENCE-LINES FOR AXIAL FORCES IN CHORD-MEMBERS.

Figs. 6.



VALUES OF MAXIMUM AND MINIMUM MOMENTS IN CHORD-MEMBERS.

the variation in maximum bending-moment and axial force in members, as the unit load travels from end to end of the girder. It will be noticed that in the Tables the values of α and β are given from one end to the centre, and that in the influence-line diagrams the values of α or β are plotted from end to end of the girder. This is due to the fact that α and β are connected by the relation

$$(\alpha_r)_R = -(\beta_{N-r+1})_{N-R}$$

where $(\alpha_r)_R$, $(\beta_r)_R$ are the terminal couples acting on the rth chord-section, when the unit load is acting at the Rth panel-point.

Diagrams are also given showing the maximum and minimum moments in each panel, due to a continuous load of uniform intensity travelling across the girder.

The diagrams show, perhaps more clearly than the Tables, the effect of variation in the stiffness-ratio, and the effect of variation in the number of panels can also be inferred. In the diagrams of moments, the ordinates represent the numerical coefficients given in the Tables, and the values of the moments are proportional to the lengths of these ordinates multiplied by $PL/8$, where P is the panel load and L is the panel length in each case. Using this scale the bending-moment diagrams are of the same order of magnitude for different values of N, so that, if a girder of definite length has to carry a definite load, the moments will be approximately proportional to the square of the panel-lengths. A similar comparison of the total axial forces in the chords shows that they are virtually unaffected by the panel-length, hence a large number of panels giving a small panel-length is likely to ensure an economical chord section.

It is, however, unlikely for a panel-length much less than the depth of the girder to be economical, both from the consideration of increasing the number of posts and of increasing the stiffness of the chord-sections by diminishing their length.

The Girder of Non-Uniform Section.

When the chords are not parallel, and chord-members are of different stiffness, particular solutions might be obtained by the application of the general principles outlined on pp. 70–71, but the results could not be presented in convenient forms such as are obtained for the girder of uniform section. It is, moreover, doubtful whether the complete analysis of such cases is necessary, since the distribution of moments and forces in girders of reasonable stiffness must be closely related to the distribution in the corresponding girder

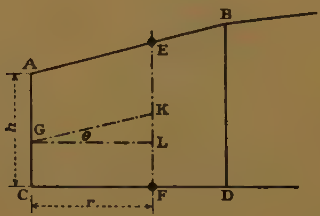
with rigid web-posts. The problem is perhaps most usefully considered in terms of the positions of the points of inflexion in the chord-members. In Table XI are given the values of x in equation (32), that is, the position of the points of inflexion, for a girder of ten panels with the three different values of the stiffness-ratio, and for a fully-loaded girder in each case.

TABLE XI.

Panel Number . .	1	2	3	4	5
Case 1, $1/s = 0$	0·500	0·500	0·500	0·500	0·500
Case 2, $s = 1$	0·449	0·533	0·564	0·610	0·833
Case 3, $s = \frac{1}{3}$	0·417	0·576	0·675	0·822	1·488

It has already been shown that the axial forces in the chords do

Fig. 7.



not differ much as between cases 1 and 2, but the bending-moments are subject to greater variation. Now the values of x in Table XI give a direct indication of the ratio of positive and negative bending-moments in each panel, but these bending-moments are due to the shearing force, which is much less at the centre than at the ends of the girder, and their accurate estimation is therefore more important in the end than in the centre panels. It is seen from the Table that in the three end panels the greatest divergence of the values of x between cases 1 and 2 is about 10 per cent., and it is not unreasonable to infer that, for intermediate values of the stiffness-ratio, the value of x could be estimated closely enough for the purpose of practical design. Values of x for a moving load, corresponding to those given in Table XI, can be found by using, in equation (32), the appropriate values of α, β as tabulated in Tables II to X. When the values of x have been estimated, the approximate values of the moments can be found, for example, by taking a section through the points of inflexion in the panel (*Fig. 7*).

A further assumption is necessary, namely, that the bending-

moments in the upper and lower chords are in proportion to their stiffnesses, and although it has been shown that this assumption is not strictly true it will give a good approximation except in extreme cases.[1]

In *Fig. 7*, ABCD represents the end panel of the girder. E and F are the points of inflexion found by estimating the values of x, and AC is divided at G so that the ratio AG : GC is equal to the ratio of the stiffnesses of AB and CD respectively. Then if EF is a measure of the total shearing force in the panel, KL represents the part carried as direct stress in AB, and EK and LF represent the shearing forces in AB and CD, approximately. Thus, if S denotes the total shearing force,

the shear in the upper chord is $$S \cdot \frac{s_1}{s_1 + s_2} \cdot \frac{h}{r \tan \theta + h},$$

and the shear in the lower chord is $$S \cdot \frac{s_2}{s_1 + s_2} \cdot \frac{h}{r \tan \theta + h},$$

where s_1 and s_2 denote the stiffnesses of the upper and lower chords respectively, and h, r, θ are as shown in the diagram. Then the fraction $h/(h + r \tan \theta)$ gives approximately the ratio in which the moments in each chord are reduced by the inclination of the upper chord.

The Paper is accompanied by four diagrams in the text and three sheets of diagrams, from which the Figures in the text have been prepared.

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[1] See footnote 2 (*b*), p. 67.

The Council invite written communications on the foregoing Paper, which should be submitted not later than three months after the date of publication. Provided that there is a satisfactory response to this invitation it is proposed, in due course, to consider the question of publishing such communications, together with the Author's reply.

Paper No. 5024.

"Earth-Pressure on Flexible Walls."

By Jens Peter Rudolf Nielsen Stroyer, M. Inst. C.E.

(Ordered by the Council to be published without oral discussion.)

TABLE OF CONTENTS.

Introduction.

One of the less exact sciences to-day is that dealing with the properties of what is commonly called "earth." Considering the extremely wide range of the term, there is perhaps no cause for surprise, especially when the heterogeneousness of most of the materials considered is taken into account. Compared with most other materials with which the engineer has to deal, and of which the properties have been thoroughly investigated and are intimately known, there is a scarcity of information available on the various problems presented by, for example, the study of lateral pressure.

One of the most original of the individual workers in this connection is Dr. K. Terzaghi of Vienna, the results of whose research have, in many respects, revolutionized current ideas. His experiments on the earth-pressure exerted on fixed and movable walls are based on entirely new methods, as ingenious as they are simple and direct. The manner in which the "at rest" value of the earth-pressure is determined is novel and very simple, and by his method this value has definitely been found. The fact that even the slightest movement of the wall, against which it is desired to measure the earth-pressure, influences the latter in a very marked degree, at once renders ineffective all devices that rely on the wall itself to register the pressure against it. This difficulty has long been realized, and as far back as 1890 Professor A. Donath endeavoured to determine the

" at rest " value from a series of measurements on the variations of pressure on a movable wall, correlating the movements and corresponding pressure values, and deducing from these results the value of the earth-pressure on the wall " at rest." The value found in this manner gave a liquidity-factor (which is represented by f in the expression $p = whf$) of 0·26 for sand with a slope of $33\frac{2}{3}$ degrees and a weight of 100 pounds per cubic foot, and this agrees fairly well with the Rankine value for f, namely $(1 - \sin\phi)/(1 + \sin\phi)$. Dr. Terzaghi's positive method gives, for similar conditions, a liquidity-factor $f = 0{\cdot}42$, which is considerably higher than any value previously found or used. This may be due to the fact that the value had never before been determined under true " at rest " conditions.

Fixed and Movable Walls.

The relations between the movements of a wall and the pressure exerted by a fill against it do not appear to have received serious attention before Professor Donath's experiments. In 1882 Mr. G. H. Darwin [1] put forward the results of experiments with sand fill in a box, one end of which could move outwards. He measured the sand-thrust by a spring device actuated by the door itself, and found the force required to hold the wall against the fill, at the moment when the wall began to move away from the fill, " in little jerks," as he described it. Dr. Terzaghi also carried out experiments on movable walls some years ago.[2]

Generally speaking, it may be said that the earth-pressure decreases when the wall moves away from the fill, and increases when the movement is towards the fill; in the latter case the form of earth-pressure which is usually termed " passive " is produced. It is difficult to ascertain the nature of the actual movement of the particles in the soil itself during a slight movement of the wall. Dr. Terzaghi gives an explanation of the happenings in the fill-particles, based partly upon mathematical reasonings on the statical resistance of a heap of spheres, and partly upon some experiments on the grouping of flat pieces of aluminium cut to the exact shape of highly-magnified sand particles. These flat aluminium bodies were confined between two glass plates, and their movements under various angles and forms of restraint were observed, their two-dimensional behaviour being used as a basis for the investigation of three-dimensional movements.

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1 " On the Horizontal Thrust of a Mass of Sand," Minutes of Proceedings Inst. C.E., vol. lxxi (1882–83, Part I), p. 350.

2 " Old Earth-Pressure Theories and New Test Results," *Engineering News-Record*, vol. 85 (1920), p. 632.

The Author, in a Paper[1] in 1927, advanced the opinion that as soon as the wall begins to move either outwards or inwards, shear forces will come into play in order to preserve the equilibrium; at first there are elastic deformations of the particles themselves, but soon the increased movements of the wall are followed by minute sliding and rolling of the separate particles. This readjustment of the particles is being performed under the restraint of the various stresses produced, and the soil is standing in equilibrium, but under conditions of stress. If, however, a state of flux can occur, during which the particles can move freely, then all the shear forces, and the stresses under which the soil had been standing previously, disappear, and the soil returns to normal conditions. This free movement of the grains is different in character from the small restrained movements which take place during the movement of the wall under stress conditions, and the soil, both during and after this free re-grouping, would appear to be in a similar condition to that of soil freshly filled-in, namely, exerting a pressure of the same order as the original earth-pressure.

The form of movement discussed above, with which all the above experiments are dealing, is the movement of a stiff wall which either tilts or moves parallel to its original position. Mr. Darwin's wall was hinged at the bottom of the box, whereas Professor Donath's wall had a semi-parallel motion, being hinged at a point well below the bottom; Dr. Terzaghi's wall moved, on rollers, parallel to its original position. In none of these cases is the movement a deflection in the usual sense of the word, inasmuch as the wall itself is purposely made as stiff as possible with a view to eliminating any wall-deflection.

Flexible Walls.

The special problem to which the Author wishes to draw attention is connected with phenomena caused by the elastic deflection of retaining-walls, and is essentially different from that illustrated by the examples above.

When a retaining-wall suffers elastic deformation from the pressure of the fill, conditions are different from those represented in the experiments above. The movements are no longer performed under constant pressure, but with an increasing resistance from the wall-material. In addition to this essential difference, there are accompanying phenomena of friction, which are totally absent in the cases investigated previously. Both causes tend to reduce the pressure on

[1] "Earth-Pressure on Flexible Walls," Minutes of Proceedings Inst. C.E., vol. 226 (1927–28), p. 116.

the flexible wall,[1] but although this circumstance is known and has been taken advantage of for years, it appears to be based mainly on theoretical considerations and on the fact that, in practice, thin retaining-walls between two supports stand up to far more than calculations would permit. At the time of reading his previous Paper,[2] the Author felt that the absence of experimental data was a serious drawback to any rational discussion of these problems, and he decided to carry out a series of tests with an apparatus already designed, but not yet tried, and with a view to ascertaining some of the data most urgently required. Since then, a great many tests have been carried out with sand and other materials.

Some of the assumptions put forward in the Author's first Paper have been confirmed by these experiments, while others have had to be modified slightly.

These assumptions were :

(1) A definite decrease occurs in the bending-moment on a wall that flexes outwards between two supports.

(2) The reduction is a function of the liquidity factor

$$f = \frac{1 - \sin \phi}{1 + \sin \phi}.$$

(3) The reduction is a function of the magnitude of deflection, depending on the wall thickness.

(4) The reduction does not apply to the reactions, but is a redistribution of pressure only.

(5) Slides may occur with great deflections, and constitute a state of flux.

(6) During a state of flux the pressure will revert to normal from the stress-condition under which the soil is standing, the pressure being either decreased or increased in the process.

While the tests seem to have proved that (1), (2), (4) and (6) are undoubtedly true as they stand, on the other hand (3) does not seem to apply, at least not in the sense in which it was suggested ; further, (5) is only correct when the qualification is made that slides, when they do occur, are more in the nature of slips, and never succeed in raising the pressure fully to the normal state, as anticipated.

What the tests did prove was that, during flexure, there was a definite reduction in the bending moment with all the five materials tested, and on all the six wall-thicknesses used in the tests. The

[1] See "Concrete Structures in Marine Work" by R. Stroyer, London, 1934, pp. 24–25.

[2] See footnote 1, p. 96.

results also proved that this reduction seemed to be a function of f; although at present no very definite relationship can be formulated between the reduction-factor r and the liquidity-factor f, r appeared to be in the neighbourhood of $2f$ under the conditions prevailing in the testing apparatus. This reduction in bending moment was found to be independent of the magnitude of the deflection and therefore of the wall-thickness, although the latter appeared to influence the value of r slightly for an entirely different reason, as will be shown later. During and after flexure, the soil is in an abnormal stressed condition of reduced pressure, referred to as the "flex" condition, and if a state of flux can occur, the pressure reverts to the normal condition. This also applies to the opposite condition of increased pressure, such as is engendered by the wall moving towards the fill, with passive earth-pressure, as a state of flux will cancel this pressure condition and produce normal pressure. The "flex" condition is generally stable, as a state of flux cannot readily occur in practice except in certain types of bunkers or silos with side discharge. The slides that occurred in some of the tests are an arrested state of flux during which the pressure rises suddenly, but the duration of the slides is so short that, generally speaking, the interference with the wall-stresses is too small to be serious. The circumstances governing the occurrence of slides appeared somewhat puzzling at first; the tests on sand may be taken as an example, in regard to which the late Professor Krey said that, in his opinion, such a rise in pressure, brought about by a slide, could only occur if the soil were damp and cohesive, or if the wall were made to move outwards in jerks. The control of the movements of the wall, as will be shown later, precluded jerks of any description, and slides did occur with dry sand but never with moist material. Further, in the series of experiments with sand, they occurred with thin walls but not with thick ones. With Kentish rag dust, which was another test material, they never occurred under any circumstances. With pea gravel there were distinct slides with the thinner walls, as in the sand tests, but they disappeared in the course of some years, with the same material in use. On the other hand linseed, a material chosen because of its high liquidity-factor, namely $\phi = 24$ degrees, showed no slides, even on the thinnest wall, on the first day of the tests, but within 3 weeks had developed the heaviest slides of all materials, and with all the wall-thicknesses.

As the Author has only been able to devote part of his time to these tests, they have extended over several years. Each test needs a considerable amount of preparation; a filling requires about thirty loads of material weighing about 100 pounds each to be hoisted 10 feet and carefully placed and strickled off horizontally in the test-

box, and takes two men nearly 2 hours, while the test itself generally occupies an hour. The steel plates used as flexible walls weigh from 100 to 150 pounds, and changing them is also a long and tedious process, which had to be carried out a great many times. A number of preliminary tests were necessary in order to perfect the apparatus and to eliminate sources of misreading, and a great many more tests would be necessary in order to study the influence of a number of factors which undoubtedly have some bearing upon the question, such as:

(*a*) The application of either positive or negative surcharge.

(*b*) The method of placing the fill-material; namely either horizontally or in layers sloping either up or down, and also whether consolidated or trickled into the box.

(*c*) The moisture content of the material.

(*d*) The nature of the flexible wall, whether made of steel, timber or reinforced concrete.

(*e*) The position of the wall; namely, either vertical or sloping forwards or backwards.

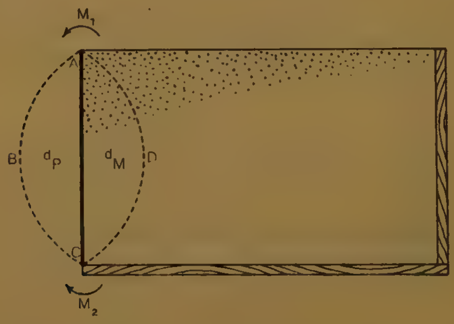
Nature of Tests.

The experiments have been carried out in order to meet two main lines of inquiry, as the flexible walls generally found in practice fall into two groups. The wall may either be a "free wall" built in the open and where the fill is brought in after the completion of the wall, or it may be a "sunk wall," where the fill is in existence before the wall is built; an example of the latter is when a sheet wall is driven into the ground, and afterwards bared in front by excavation or dredging, the wall deflecting during the operation. In the latter case the whole height of the fill is present during the flexure of the wall, and the factors producing the reduction in pressure, such as, for instance, double friction, are present to their full extent, whereas with the "free wall" this is not the case and the wall flexes while the fill material is brought in, so that the factors mentioned above would not appear to attain their full value until after the filling had been completed. It may be said at once, however, that, allowing for various other factors inherent in the different methods of testing these two cases, there did not appear to be very much difference between the two types of wall. Approximately the same number of tests have been carried out on the two kinds of wall.

The apparatus, to be suitable for the two conditions, must be able to let the flexible wall deflect freely, and also to imitate the sunk walls, where the plate is kept stationary during filling and afterwards allowed to deflect under control. In practice, when a sunk wall is being

constructed, the pressures at the start are equal and opposite on both sides of the wall, which is stationary until the dredging operations begin. The baring of the front of the wall deprives the latter of the resistance from the front, and the wall is called upon to keep up the back-fill to an increasing degree by its own strength until, at the end of the excavation, all help from the soil in front has disappeared, and nothing but the wall-resistance itself remains to retain the back-fill. During the excavation the wall flexes, until at the end it stands with a deflection corresponding to the earth-pressure behind it. Whether this earth-pressure is the same as, or smaller than, that existing before the dredging started it is difficult to prove in practice, since there is no means of determining the value of this initial earth-pressure. The evidence from practice goes to show, however, that the final pressure on the wall after excavation must

Fig. 1.



be considerably smaller than the ordinary calculated value. In the apparatus it is possible, as will be seen, to determine the pressure on the wall both before and after the dredging or excavation.

The design of the apparatus for allowing the wall to deflect under control is based on the following principles: let AC (*Fig. 1*) represent a flexible wall supported at top and bottom, and forming the end of a box in which filling-material may be placed in order to exert lateral pressure on the wall. Under the influence of this lateral pressure the wall will deflect outward to the position indicated by the line ABC. Moments M_1 and M_2 applied at A and C will deflect the free wall to the position shown as ADC. It is possible to adjust the moments so that the flexure due to them cancels that due to the fill pressure, in order that the wall will stand without flexure. The magnitude of the moments required to balance the earth-pressure may easily be found by equating the deflections d_P and d_M. The relative magnitudes of M_1 and M_2 are determined in order to give

the maximum deflection at the same point, namely, at a depth of 0·519 of the span AC, as that due to earth-pressure, assuming the usual triangular loading. This will be the case when M_2 is equal to $1{\cdot}6M_1$. Then the deflection d_M due to the moments M_1 and M_2 is

$$d_M = 0{\cdot}1628\,\frac{M_1H^2}{EI}$$

where H, E and I are the span AC (36 inches in the apparatus), the modulus of elasticity and the moment of inertia of the wall.

The deflection due to the earth-pressure P is

$$d_P = 0{\cdot}01304\,\frac{PH^3}{EI}$$

and by equating these expressions,

$$M_1 = 0{\cdot}08\,PH.$$

In the apparatus it is possible to apply the moments M_1 and M_2 gradually and in the correct proportion as the filling of the box proceeds, so that at the end of the filling-process the wall stands without flexure along the line AC and with the full earth-pressure behind it. This pressure may be found from the moments M_1 and M_2, the value of which can be measured directly, as

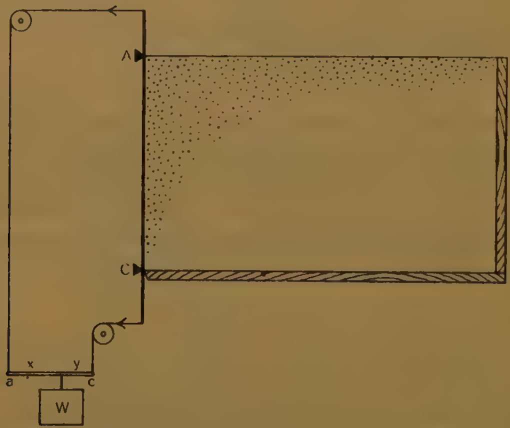
$$P = 12{\cdot}5\,\frac{M_1}{H}.$$

This is the actual earth-pressure at the beginning of the test, corresponding to the value acting on the sheet-piling before dredging begins. The counterpart of the dredging is the decreasing of the moments M. If these are allowed to decrease under control, the wall will begin to flex outwards, and if the earth-pressure P keeps constant during this decrease of M, the deflection at any time should be a function of M only, and be proportionate to the decrease in M. When this decrease has reached its full value, namely when M has decreased to zero, the outward deflection should be d_M or d_P, since M and P have been balanced to give equal and opposite deflections at the start. Should it be found, however, that at the end of the test, or at any time during the decrease of M, the actual measured deflection of the wall is not the calculated function of M, then the earth-pressure has not kept constant, and a loss in earth-pressure is shown as a loss in deflection.

The wall may be kept stationary during filling by other methods than that of applying the moments M as described above, and several ways were tried during the tests, and will be described later. The final balancing was, however, always obtained by means

of the moments M, which were produced in the apparatus by cantilevering the vertical wall beyond its supports A and C to an extent of 9 inches, as shown in *Fig. 2*. A weight W hung from a crossbar ac exerted a horizontal pull at the end of the cantilevers, the weight being so placed on the crossbar as to give the two moments their desired relative value, namely x equal to 1·6 times y. The weight W consists of a large drum that may be filled with water through a hose connected to a tap, and emptied in a similar manner, so as to give an easy and gradual adjustment without shock or vibration. The height of the water in the drum, and consequently the value of W, is registered by means of a scriber connected to a

Fig. 2.



float in the water-drum. The value of the balanced earth-pressure P can be found from the expression

$$P = 12{\cdot}5\,\frac{M_1}{H}$$

by substituting $\frac{9\,Wy}{x+y}$, or $\frac{9\,W}{2{\cdot}6}$ for M_1,

so that $$P = \frac{12{\cdot}5 \times 9W}{2{\cdot}6 \times 36} = 1{\cdot}2\,W,$$

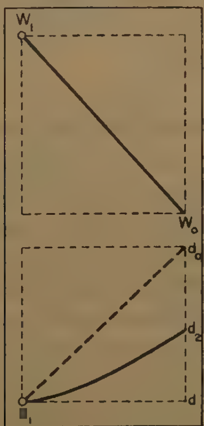
where W and P are the weight of the water and the earth-pressure on the 36-inch wide plate, respectively.

In order to facilitate the reading of the corresponding values of the moments M, that is, of the weights W, and the deflections d, the time element is introduced, so that W and d are scribed on a rotating drum during the outflow of water.

If the outflow of water takes place at a constant rate, the W-line

on the drum record becomes a straight line when developed. The d-line due to this variation in M or W is also a straight line, and represents the deflection-line for an earth-pressure that has kept constant during the outflow. In other words, this d-line is the check-curve for the behaviour of the earth-pressure during the outflow test; if the deflection actually recorded is greater or smaller than that represented by the check-curve, the earth-pressure has been greater or smaller than the value at the beginning of the test. This check-curve may be calculated from the characteristics of the wall, but it is simpler to let the deflectometer scribe this line during a check-test, in which the wall-plate alone is unloaded at the same rate, after having been loaded up to the same weight W as was necessary to balance the earth-pressure. Referring to *Fig. 3*, which represents

Fig. 3.



the development of a drum-record, the upper line W_1W_0 indicates the variation in W during the emptying of the drum, from the balancing value of W_1 inches of water at the beginning of the test, to W_0 inches of water at the end of the test when the drum is empty. The check-curve for the plate alone, during this emptying, is d_1d_0, found by a check-test as described above. If the actual deflection during the earth-pressure test is a line d_1d_2, then the earth-pressure at any point is smaller than the starting value, and at the end is smaller by the amount represented by $d_0 - d_2$. The total free-plate deflection $d - d_0$ corresponds to a weight of W_1 inches of water, equivalent to an earth-pressure $P = 1{\cdot}2\ W_1$, and the loss in earth-pressure during flexure is therefore

$$P\frac{d_0 - d_2}{d_0 - d},$$

the actual (reduced) pressure at the end of the test being

$$P\frac{d_2 - d}{d_0 - d}.$$

Instead of letting the deflectometer scribe the check-curve, it is only necessary to find the deflection of the free plate due to one inch of water in the drum. As the apparatus is so designed that a mark can be made on the record for every inch the water drops in the drum, all that is required to plot the check line is to mark the record per inch as described throughout the test, and then set off the plate deflections per inch of water. In this way the speed of the outflow is of no consequence, and this is the method that has been followed, as it was found that a great many complications arose in connection with the discharge of the water at an absolutely constant rate. The outflow, however, takes place through a long hose discharging at a much lower level than the bottom of the drum, so that the check-curve is very nearly straight.

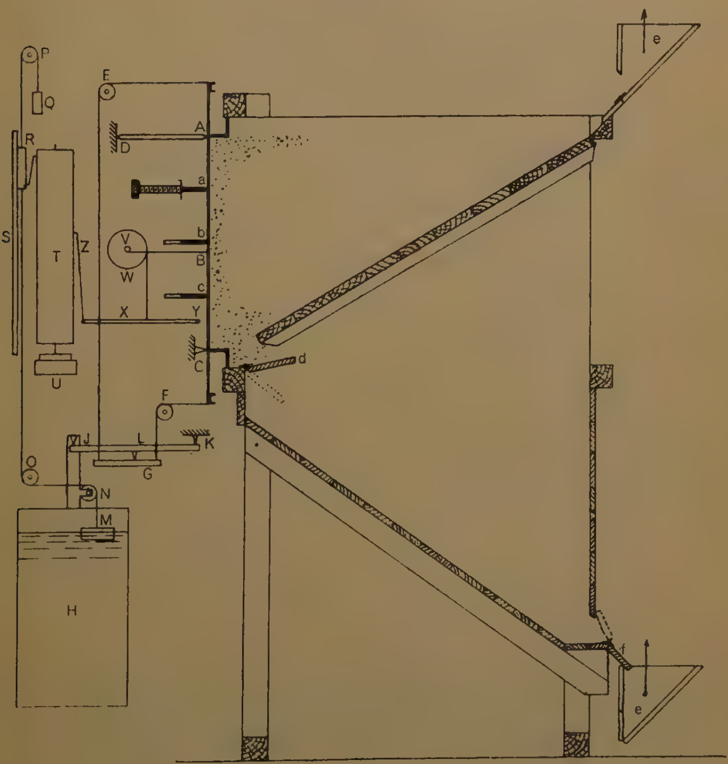
Experimental Apparatus.

Referring to *Fig. 4* (p. 105), which shows diagrammatically the general arrangement of the test apparatus, ABC is a mild steel plate, cold rolled and hydraulically stretched, supported at C on a fixed horizontal knife-edge, and at A on a pendulum knife-edge, fulcrumed at D. This movable support was found necessary owing to the great curvature, and consequent alteration in span, in some of the thinner plates tested. The plate is cantilevered out 9 inches beyond the 3-foot span, and strengthened at the ends by a small channel, to the centre of which a chain or flexible wire is attached, and carried over ball-bearing pulleys E and F down to the crossbar G. In the later tests bell-crank levers on pin-point supports were substituted for the pulleys. The weight from the water-drum H acts on the crossbar G at a point L, determined by the distances x and y which were dealt with on p. 102. The drum H is 18 inches in diameter and 30 inches deep, and its weight on the crossbar is doubled by a lever suspension, H hanging on a knife-edge at the end of a lever J fulcrumed on another knife-edge at K, and pressing on the crossbar with a steel point L. Every inch of water in the drum adds a weight of 9·2 pounds to the drum, which acts on the crossbar as 18·4 pounds, and is equivalent, for deflection purposes, to an earth-pressure of 22·08 pounds on the plate. From the float M in the drum, a flexible cord is taken horizontally over the compensating pulley N, and vertically between the pulleys O and P, a counterweight Q holding the cord taut. The scriber R is attached between O and P, guided

by the steel plate S and recording the float movement on the rotating drum T, which is driven by the clockwork mechanism U.

Several types of deflectometer were tried, the most satisfactory being the one illustrated, inasmuch as it is positive and easily altered to give different magnifications. Attached to the plate is a strong thread that winds round the $\frac{1}{8}$-inch-gas-screwed spindle V, which runs in ball-bearings and carries a sheave W. Another

Fig. 4.

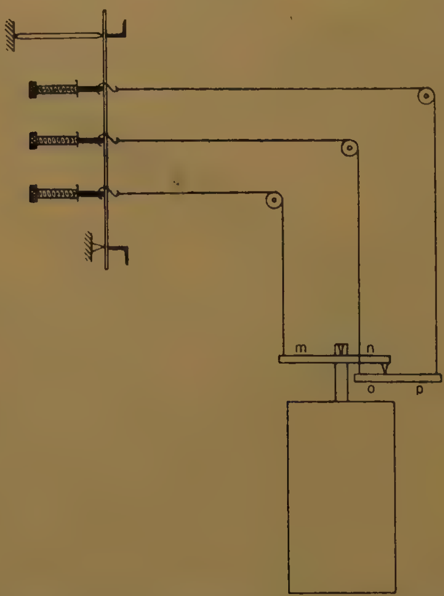


thread is attached to and runs over the sheave, and actuates a light lever X fulcrumed on two pin points at Y, which carries at its other end the deflection-scriber Z; this records the plate movements on the drum T. The deflectometer and the float recorder scribe at opposite sides of the recording drum, and the magnifications employed for the different plates vary from about 10 to 100.

Instead of balancing the earth-pressure during the filling of the box by applying moments M to the plate as described above, the

plate may be kept stationary in other ways, one being the introduction of straight-edge supports a, b and c between the main supports A and C. These straight-edges can move forwards and backwards in slots, and may be adjusted by means of finely threaded thumb-screws bearing on the straight-edges through thin metal feelers. Before the filling is introduced, the straight-edges are brought forward so that they just touch the plate, and the thumb-screws are screwed up to the feelers so that the latter are just held in position. At the end of the filling process the feelers are held tightly, and water is then run into the drum and the weight

Fig. 5.



adjusted to balance the earth-pressure, so that the feelers are again just held in place. The thumb-screws holding the straight-edges may then be loosened and the latter removed sideways through the slots.

Still another method of balancing the earth-pressure is by applying the weight of the water directly to the straight-edges mentioned above. *Fig. 5* shows the arrangement for employing this method, in which the weight, acting through a system of levers, may be adjusted to give the correct pull at the three straight-edges, so that they loosen from the feelers at the same time.

Of the three methods the direct balancing, without straight-edges, was found to be the easiest. In order to keep the plate stationary

while the back-fill is being placed, the deflectometer must be carefully watched while manipulating the inflow of water to the drum, as it is too late to send in water-ballast after an outward deflection has taken place. After a few trials, however, the movements of the deflectometer during the charging of the box could be anticipated, and it became a comparatively easy matter to regulate the water-supply accordingly. If straight-edges are used, equal care is required to stop the water-supply just at the moment when the feelers begin to loosen. It may be mentioned that the balancing value, that is, the number of inches of water in the drum which represented the actual earth-pressure, was found to be the same with all three methods.

The box is made of stout planking, and the plate forming the flexible wall has a span of 36 inches, while the width of the box is also 36 inches and the length 5 feet 6 inches, the bottom being sloped at an angle smaller than the least angle of repose of any of the materials tested. In this manner the amount of filling material is reduced, and the tedious process of filling the box is shortened. A normal filling requires about thirty skip-loads of 100 pounds each to be raised about 10 feet, and takes nearly 2 hours. The fill may be unloaded from the test-box through a hinged flap d at the lower end of the bottom, the flap being manipulated from the front of the apparatus. The filling material discharges from the test-box into the storage-space below; it is kept there until required for another test, when it is made to run into the skip e by opening another flap-valve f. The skip may be raised to a position above the test box by means of a pulley and counterweight, and is of such a shape that it automatically discharges its contents into the test-box at the end of its travel.

The width of the steel plate is slightly less than that of the box in order to allow the plate to move freely when it is deflecting, and a movable joint between the plate and the sides of the box was made by using strips of thin felt bent into an L shape. One wing of the "L" was fixed to the side of the box along the joint and the other rested against the plate, so that the pressure of the fill itself kept the joint tight, in the same manner as a hydraulic packing. A tight and yet flexible packing was thus obtained along the two sides and bottom of the box.

Steel Plates.

A number of steel plates of different thicknesses have been used in these experiments with a view to testing the effect of the magnitude of the deflections, especially when the latter become excessive. The plates are hydraulically straightened so as to remove any buckles.

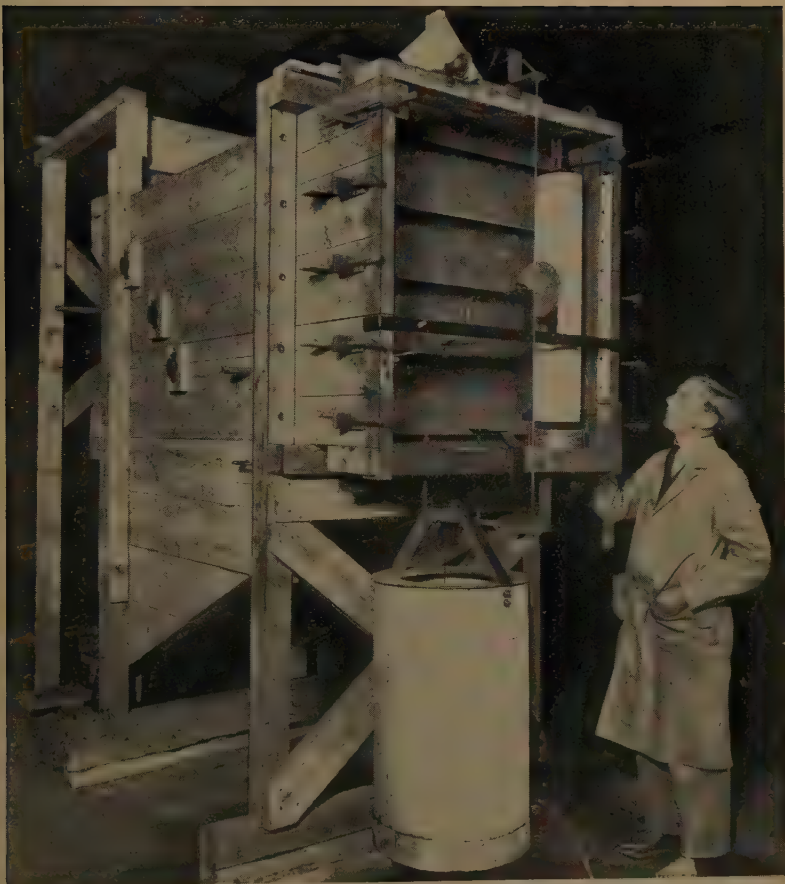
The plates were 3 feet by 4 feet 6 inches, and not being of a standard size, it was found that during guillotining the straightness was destroyed, and it became necessary in all cases to have the plates flattened by hammering them. This is a slow and expensive process, and is almost a lost art, but is absolutely necessary in view of the fact that the slightest buckle in the plate gives discontinuity in the check-curve, while the jerk may also start slides in the fill behind. The change from positive to negative flexure must take place absolutely gradually and without the slightest jerking in the deflectometer record. Six plates have been used, measuring from 0·09 to 0·25 inches in thickness, particulars of four of them being as given on p. 129.

The photograph, *Fig. 6*, is a view of the testing machine which at the time had been used for several hundred tests and was showing signs of wear and tear. By comparison with *Fig. 4* the arrangement is self-explanatory. The deflectometer-sheave shown on the machine is one giving a magnification of about twenty-four. All rotating spindles are fitted with ball-bearings, and all rocking bearings are pin-point or knife-edge suspensions. The hammer-marks from the straightening process are plainly visible on the steel plate.

Filling Materials.

Five filling materials have been used in these tests in order to ascertain the influence of the liquidity-factor *f*. While for this purpose it was desirable to employ materials whose angles of repose covered as wide a range as possible, it was found that in actual practice this range was limited to between approximately 24 and 40 degrees. The latter limit may appear to be too low, for most text-books, including the Author's, refer to slopes up to 50 or 55 degrees, and such ordinary materials as coal and coke will immediately come to mind. It is true that both these materials will stand with slopes greatly in excess of 45 degrees, but this steep slope is not a natural one, and once the equilibrium is disturbed there is a landslide which leaves a slope of approximately 40 degrees. The testing machine is large enough to deal with these materials without introducing any wedging between the supports, the span of the plate being 3 feet; however, apart from the fact that they are rather dirty materials to use in the machine, where the continual loading and unloading gives a great deal of dusting, their natural slopes are no steeper than that of granite chippings, which was ultimately found to be the steepest filling material available. This has an average slope of 40 degrees. In the search for steep slopes a supply of Kentish rag-

Fig. 6.



TESTING MACHINE.

dust was obtained, as this seemed able to stand up very steeply. Anyone who is familiar with this material will know, as the Author found, that under certain circumstances it will stand almost vertically, and in some tests, where a certain amount of consolidation was introduced, it even formed an overhanging slope; as in the case of coal and coke, however, these steeper slopes are unstable and break down on being disturbed, until a natural slope is formed. It is difficult to ascertain this natural slope, as the material is willing to stand at almost any slope given to it; as a test material it was not very desirable.

The particulars of each material are given later, with the test results, from which it will be seen that the flattest slope obtained in any of the materials tested was 24 degrees; it would no doubt be possible to have still flatter slopes with semi-liquid muds or silts, but as the apparatus was not designed to hold wet fillings, the Author had to be content with this minimum slope. None of the ordinary dry filling materials have a sufficiently high liquidity-factor to give such a flat slope, and, after many trials on unsuitable materials, linseed was found to be the best for this purpose. It is not, strictly speaking, a filling material, but it may be stored in bunkers, and its pressure-characteristics throw light on several of the factors entering into the questions under discussion. Linseed is almost perfectly granular in form and is a very lively and liquid material; it weighs 40 pounds per cubic foot, and stands with the flattest slope so far obtainable. It promised to be a most suitable test material, but was soon found to alter its characteristics at such a rate that the Author could not follow up quickly enough with the tests. The latter, as will be understood from the foregoing description, take some time to prepare and carry out, and it was not possible to arrange for more than three tests on the first day when the linseed arrived; during the time taken by these tests the linseed had changed its characteristics from a slope of 24¼ degrees and a weight of 40 pounds per cubic foot, to 26 degrees and 38 pounds, the corresponding fill-pressures altering from 185 pounds to 160 pounds respectively. In the three weeks it took to test the linseed on all the plates, it changed further to 33 degrees slope and 36 pounds weight, the fill-pressure meanwhile falling to 118 pounds. As the material is somewhat expensive (the quantity required cost about £10), only the one set of tests has been carried out, but sufficient data have been obtained to illustrate the variations in the fill-pressure entailed by these changes, and for this reason the latter have proved most instructive. The other two materials used during the tests were sand and pea-gravel.

Description of Tests.

A test on a wall of the "Free" type is the easiest to carry out, and consists simply in measuring the deflection of the plate while the fill material is brought in behind it. Every few skiploads of material are carefully brought forward against the plate in horizontal layers and strickled off, avoiding, as far as possible, shocks or vibration, until the filling is completed with a level surface behind the support A. Neither the water-drum nor the clockwork is required for this test, the recording drum remaining stationary while the deflectometer scribes a vertical deflection-line. The latter is marked at the commencement and at the end of the test, and may also be marked at any intermediate point while the filling is proceeding, such as at every 3 inches of filling. The earth-pressure corresponding to any degree of filling may be obtained from the deflection at any point, either by calculation, when the characteristics of the plate are known, or from the check-tests on the plate, thus giving its deflection per inch of water. If the total deflection is d_P and the deflection of the plate per inch of water is d_W, then the recorded earth-pressure corresponds to d_P/d_W inches of water, and its value in pounds is found as

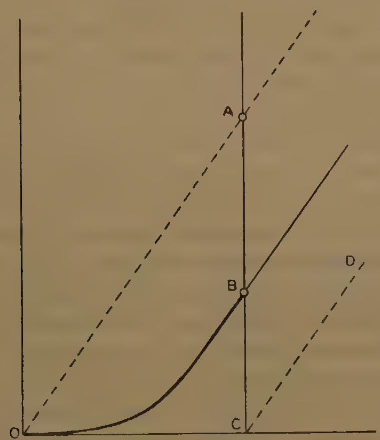
$$P = 22{\cdot}08\,\frac{d_P}{d_W}.$$

The term "inches of water" will be used in references to the various test results, it being remembered that one inch of water in the loading drum represents 22·08 pounds of earth-pressure on the plate. For comparative purposes in these tests, the "inch of water" will be found the most convenient unit.

To test a wall of the "sunk" type, the plate must have remained stationary while being back-filled, before the test can commence. This is accomplished by one of the methods already described, and when the plate has been perfectly balanced the weight of water, W_P, in the drum gives the actual earth-pressure at the beginning of the test. The clockwork for the recording drum is then started and the deflectometer allowed to scribe a horizontal starting-line. The water-outlet from the drum is then opened, and the time for the start of the test simultaneously marked on the record. As the water sinks in the water-drum, the plate deflects and the deflection-indicator scribes a line on the record, a mark being made for every inch of water emptied. At the end of the test, when the drum is empty, the deflectometer is made to scribe another horizontal line representing the final deflection. The record may then be removed and the check-line drawn in and compared with the actual deflection-line. A typical record is shown in *Fig.* 7, where the check-line OA

has been drawn dotted, while the record OB is a full line. The deflections thus recorded are always smaller than the check-deflections, and the deflection-line clings to the starting line as a tangent, before curving upwards parallel to the check-line, and at a certain distance below it. The time taken for the water in the drum to empty out can be regulated, from 2 minutes with the outlet fully open to as slow a rate as desired, by throttling the outflow. The usual time for a test has been 45 minutes; tests made with a time of outflow of 5 minutes, and also with a time of 2 hours, seem to present the same

Fig. 7.



appearance, being only foreshortened or elongated in proportion to the time.

Continuation Tests.

It is possible to continue the deflection beyond the finishing point B, at which the plate is unloaded to its previous state, namely, standing under the same conditions with regard to the moments M as it was before the test. The design of the testing-apparatus allows this experiment, which has no counterpart in actual practice, to be carried out. It would correspond to the resistance of the wall being still further decreased after the whole of the excavation or dredging had been finished. This, the Author imagines, could only take place through the deterioration of the material of the wall and the consequent weakening of the section-modulus; in steel piling this may occur through corrosion, and in timber piling on account of worms or rot, but in reinforced concrete piles it is not likely to occur through any ordinary agency, since the strength of the latter tends to increase in the course of time.

If the previous conditions included a moment M_1, already applied by an initial water-load W_1 in the drum before the fill is placed, then the whole test, with the filling, balancing and emptying of the water, may take place exactly as described above, the balancing water-load W_P being added, as before, independently of W_1. When W_P inches of water have been unloaded, the record is marked at B with a finishing line representing the normal condition, and during the further unloading of the W_1 inches of water the deflection is recorded as a continuation of OB, and still keeping parallel with the check-line OA. With some of the heavier plates a considerable initial load may be applied, and the deflection-line doubled in length, without overstressing the plate. Even the thin plates will stand some initial loading, and a great number of tests have been made in this way on all the plates.

Flex Condition.

During the whole course of the deflection-diagram the actual deflection is always smaller than the check-value, namely, the full balancing value. The distance AB, which represents the amount which the actual deflection lags behind the check-value, and which will be referred to in the following description as "the lag," is increasing from zero at the beginning of the test (at O) until the deflection-line becomes parallel to the check-line, and from this point the lag is constant, however far the deflection is carried on. Any point inside the area bounded by the check-line OA and the starting line OC indicates an earth-pressure smaller than the check-value, whereas a point above the check-line would indicate an earth-pressure greater than the check-value. The former area indicates a diluted, and the latter a condensed, state of earth-pressure.

The condition imparted to the soil during outward flexure will be referred to as the "flex condition," and differs from the normal, balanced condition in that the soil is in the diluted-pressure area; the grains are possibly in a state of looser packing, and the earth-pressure exerted on the plate, as shown by the deflection, is smaller than the normal value by the amount of the lag AB. After the lag has been reached the earth-pressure remains constant at the reduced value for any deflection, since the two lines remain parallel.

If the plate is made to deflect inwards towards the fill, the condition generally known as "passive resistance" is engendered. This condition is also abnormal in the same sense as the "flex condition" described above, but in the opposite direction.

The continuation-line, recorded in cases where an initial load W_1 has been applied to the water-drum, affords a means of interpreting

the deflection-diagram. The initial loading makes not the slightest difference to the deflection-diagram proper, which is OABC in all cases; all that happens is that, when the balancing load W_P has been unloaded, both OA and OB continue parallel until the initial load W_1 has been expended. The same diagram might have been recorded by keeping the water-load constant in the drum at O from the beginning of the test, and applying negative moments M_1 and M_2 at the plate-supports (*Fig. 1*). By increasing these negative moments at a uniform rate the same deflection-diagram would be obtained. When the negative moments have grown to a value corresponding to that of the balancing load W_P (*Fig. 7*), the points A, B and C are reached, and further increase of the moments produces continuations along OC, OB and OA. If, at the beginning of the test, the soil is assumed to be transformed into a perfect liquid, with $f = 0$, the pressure on the plate will be full, constant and maintained, and will follow the check-curve OA, which is the free-plate deflection-curve. If, on the other hand, the soil is assumed to be stiffened into a solid, with $f = 1$, then the deflection line, under the action of the negative moments, would be OCD. While these negative moments increase from zero to the balancing value, the deflectometer registers nothing and scribes the line OC, and during this period the plate is simply being unloosened from the solid, until at C it is just free. After that point, the further application of negative moments bends the plate away from the solid, and at the rate shown by CD, parallel to OA.

The Lag.

From this train of reasoning it would appear that, for similar conditions of unloading, the perfect liquid behaves in the manner indicated by the line OA, and the perfect solid follows the line OCD. A soil with a liquidity factor between 1 and 0 is likely to behave in a manner between the two, and its deflection diagram will presumably be closer to OA if it has a high liquidity factor and approach OCD if the liquidity factor is low. That is, the lag will vary between 0 and 1 as some function of f, when f varies between 1 and 0. From the test results it will be seen that the lag does vary in some such manner. Taking the figures from the thinnest plate, No. 1, which are the most likely to be correct as any inaccuracies are less highly magnified, the lag l for granite chips, with $f = 0{\cdot}22$, was measured as 0·58, while for linseed, with $f = 0{\cdot}392$, it measured 0·23. For intermediate values of f, the lag had values between these two extremes. The reduction-factor r for the full earth-pressure is $1 - l$, and r should therefore vary from 0 to 1 with f. Although r appears to be somewhere near $2f$, as will be seen from the following Table this expression

is evidently untrue: it makes the reduction cease for any slope below 20 degrees, with $f = 0{\cdot}5$, and it makes $r = 2$ for $f = 1$, which is impossible.

Slope: degrees.	Material.	f.	l.	r.	$2f$.	$\frac{2f}{1+f}$	$\frac{2f}{1+f^2}$
40	Granite chips	0·22	0·58	0·42	0·44	0·36	0·42
37	Kentish rag	0·25	0·55	0·45	0·50	0·40	0·47
35	Sand	0·27	0·54	0·46	0·54	0·42	0·50
33	Linseed	0·29	0·41	0·59	0·58	0·45	0·54
32½	,, ...	0·30	0·38	0·62	0·60	0·46	0·55
32	,, ...	0·31	0·35	0·65	0·62	0·47	0·57
30	,, ...	0·33	0·33	0·67	0·66	0·50	0·58
26	,, ...	0·39	0·23	0·77	0·78	0·56	0·68

In the Author's previous Paper,[1] a value for r was suggested of the order $2f/(1+f)$, which may also be written as $1 - \sin\phi$. On this basis the value of r is on the small side, as will be seen from the Table, that is, the reductions are too drastic. If it is taken as $1 - \sin^2\phi$, that is as $\cos^2\phi$, the values of r are too high. The expression $2f/(1+f^2)$ appears to agree approximately with the test results.

As the lag appears to be independent of the magnitude of the deflection, the plate-thickness would not seem to affect its value. The lag, once it has reached its full value, is constant however much the plate deflects, but the actual value of the lag in these tests appeared to be slightly smaller with thick plates than with thin ones; this will be noticed from the tables shown later, giving the average results of a great number of tests. The reason for this is not immediately clear. The magnification of the scribed records is much greater with the heavy plates, and any inaccuracies are consequently multiplied in a higher degree than with the light plates. It is thought, however, that all such inaccuracies have been allowed for and the necessary corrections made, and a possible reason for this apparent or real decrease of the lag for the thicker plates is suggested on p. 128. Considering the difference in the plates tested, the variation in lag is small. The plates vary from 0·09 inch to 0·25 inch in thickness, and the full deflection varies from about 2 inches to ⅛ inch. Plates thinner than 0·09 inch are too weak for the earth-pressure (with reasonable deflections), while plates thicker than ¼ inch are difficult to obtain absolutely flat; the latter are also difficult to handle, as they require heavier tackle to erect and dismantle them than is convenient with the present apparatus. With a view to obtaining a stiffer plate than that of ¼-inch steel, a

[1] *Ibid.*

reinforced-concrete plate $\frac{3}{4}$ inch thick was prepared; this weighed no more than the steel plate and should have given only half the deflection, but unfortunately the hysteresis of the material interfered so much with accurate measurements that the results were unreliable and misleading. Up to the present time, therefore, the $\frac{1}{4}$-inch plate is the thickest steel wall tested in the machine.

Fig. 8.

Fig. 9.

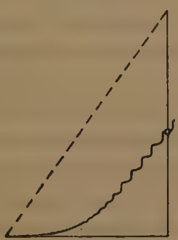


Fig. 10.

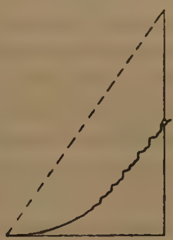
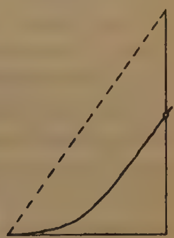


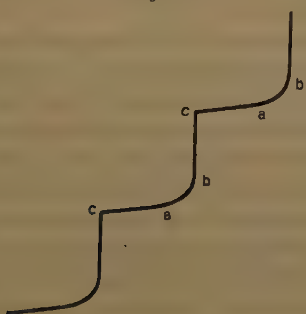
Fig. 11.



Slides.

The diagrams considered above are typical deflection diagrams. In the actual records, such as, for instance, *Figs. 8* to *11* inclusive, the parallel part of the flex line is in most cases discontinuous. This must be a result of irregular pressure-conditions in the soil developing during flexure. The external forces acting on the plate during

Fig. 12.



flexure, namely the moments M_1 and M_2, are decreased without the slightest jerk or shock; in fact, nothing could be more gradual in its application than weight controlled by the constant inlet or outlet of water; the discontinuity must, therefore, be due to pressure manifestations within the soil itself.

Referring to *Fig. 12*, where a part of the record is shown, the sudden deflection from b to c must be caused by a sudden rise in pressure. It will be shown later that a state of flux invariably

produces a rise in pressure of this nature when the soil is in the flex condition. The probable explanation of the sudden rise in pressure from b to c is that the soil during the slide, as this manifestation will be termed in the following, has been in a state of flux; that is, instead of the grains rolling and turning under control, in the parallel stretch on the record (*Fig. 11*), they have been fully released and are tumbling freely until the movement is brought to a sudden stop. While the slide is taking place, the plate is being bent outwards, its resistance increasing with its movement until the former is sufficient to stop the latter. The beginning of the slide is very gradual, as if the particles are gathering speed; the end is sudden, as if the movement has overshot its mark. The grains are probably forced into interlocking positions by the sudden stoppages or at any rate wedged into closer packing, from which it requires a certain effort to dislodge and set them moving again. This effort is represented by the unloading of water which brings about a decrease in the plate resistance during the period from c to a, in which interval the soil acts almost as a solid, until at a the particles begin to move and gather speed. Whether this movement will develop into a slide, as from b to c, or continue under control, as in *Fig. 11*, depends both on the nature of the grains and on the stiffness of the wall. Some soils never slip, no matter what plates are used, whereas, for instance, the sand in the present series of tests showed slides with all the thinner plates, but not with plate No. 4 ($\frac{1}{4}$ inch thick), no matter how far the deflection was continued.

Some sort of corroboration of the suggested grain-movements was afforded when testing the thinner plates; although generally there was too much noise in the test-shed, it was possible, on occasions when the surroundings were perfectly quiet, to hear faint sounds produced by the grains moving against the plate. The thin steel plate acts as a soundboard and magnifies any noise in connection with the plate or any object in direct contact with it. During the period corresponding to ca, no sound is heard from the plate. Between a and b, while the deflectometer is gathering speed, a faint patter may be heard, as of rain on a window, becoming faster towards b until it develops into a swish, as of emery cloth on the plate, during the rise from b to c, when sound and movement come to a sudden stop. This oral manifestation is, at any rate, not incompatible with the explanation of the grain-movements suggested above.

In the case of some filling materials, all the plates show slides, but in the case of the sand tests, the thicker plate ($\frac{1}{4}$ inch) is stiff enough to keep the movement of the particles within control, and the rate of this movement never exceeds that corresponding to the free

plate under check-load, that is, the deflection-line is strictly parallel to the check-line. Where slides occur with the thinner plates, they last throughout the whole of the deflection; once started, the slide-formation continues regularly, and the records present a fairly uniform appearance, both in respect of the time and the magnitude of the slides; consequently, the envelopes for the slides are both parallel to the check-curve. From an inspection of the records it would appear that any slight irregularity is sufficient to start a slide; the reason for the intermittent slipping is much the same as that producing "hunting" in engine-governors.

Slides, similar to those occurring in sand, appeared with granite chips and pea-gravel; but whereas in the latter case all but the heaviest plates showed slides, as in the sand tests, the slides with the granite chips were confined to plates 1 and 2, both the thicker plates being stiff enough to keep the grain-movements under control. Only four plates are shown in the Tables, as the remaining two almost coincide with two of the others in thickness and results.

The linseed started as a slide-free material, but slides developed very quickly with it, gradually became more marked, and finally were the most pronounced of all, and occurred with all the plates. On the other hand, pea-gravel, which started as a slide material, gradually lost the slides and showed slide-free records after about 2 years of use.

The Kentish rag-dust never showed slides under any circumstances, and in this respect behaved exactly like moist sand. Some tests were carried out with sand to which about 5 per cent. of water had been added, for the purpose of studying rupture-planes, and in these tests no slides ever occurred. The granite chips were at first mixed with fine granite dust, which is somewhat similar in character to rag-dust, and while mixed in this manner the material did not show slides; only when the dust was sifted from the chips did the latter exhibit slides, as mentioned above.

From these observations it would appear that the slope is not the factor that determines whether slides shall occur or not. Linseed, with a slope of 24 degrees, had no slides, neither had rag-dust nor moist sand with slopes of about 37 degrees. Sand and pea-gravel, with slopes between these limits, had slides, and granite chips, with a slope of 40 degrees, also showed them. As linseed and pea-gravel underwent a change in this respect, the particles were carefully examined, and it appeared that the only thing that could possibly have altered was the surface of the individual grains. The particles of linseed were smooth, with an almost polished surface, when it was first tested, but, as described later, the surface of the seeds became coated with dust particles which adhered firmly, and

incidentally stiffened the slope; this rough surface prevents the seeds from sliding easily on each other, and the movements are performed in jerks. On the other hand, the pea-gravel, which consists of fairly round pebbles about $\frac{1}{4}$ to $\frac{1}{2}$ inch in diameter, was rough to the touch when the gravel was first obtained, and it was noticed that the continual rubbing and rolling incidental to the charging and discharging the material during these tests, which extended for some years and were quite numerous, had worn and smoothed the surface of the individual pebbles, so that in the last tests the particles were sufficiently polished to move easily over one another without any jerks. Kentish rag-dust is a very fine powdery material, and although there is no cohesion in the ordinary sense, the particles seem to cling to each other, so that they easily form exaggerated slopes. The grains of moist sand adhere in the same manner as in Kentish rag-dust, but with the difference that in one case the grains are surrounded by a film of water, in the other by one of air. These two materials were not only alike in that they showed no slides, but they were also able to form very steep slopes, and they alone showed definite rupture planes, as will be described later.

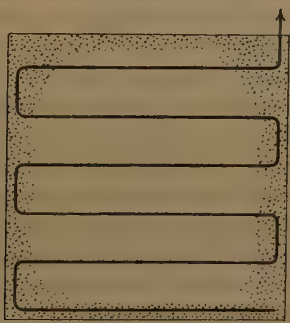
Where slides occur, the magnitude of the slide, that is, the deflection caused by it, appears to bear some relation to the strength of the plate; the lighter the plate the greater the slide. From the great number of records available, it would appear that the length of the slide is inversely proportional to the resistance-moment of the plate, or to the cube of the plate-thickness. This may be so expressed that, for the same fill material, the work done during a slide is the same with the different plates, or the deflection during a slide corresponds to the same amount of inches of water on all the plates. Thus, for sand, the deflection, or pressure-rise, equalled that produced by about $\frac{7}{8}$ inch of water. The balanced total pressure of sand corresponds to about 17 inches of water, and the flex pressure to about $8\frac{1}{2}$ inches, so that the pressure-rise during a slide is roughly 5 per cent. of the total or 10 per cent. of the flex pressure. If the average deflection curve, which is midway between the two envelopes, is taken, as has been done in reading the records, the pressure-variation brought about by the slides is seen to be $\pm 2\frac{1}{2}$ per cent. for the full pressure and $\pm$ 5 per cent. for the flex pressure, so that the practical interference with the wall-stresses is very small in this case. With pea-gravel and granite chips the slides were still smaller and, on the same basis as above, the pressure-variations only amounted to $\pm$ 1·3 and $\pm$ 2·6 per cent. (maximum) for pea-gravel, and $\pm$ 1·5 and $\pm$ 3·5 per cent. for granite chips respectively. The behaviour of linseed in this respect was different from the other materials, and from a perfectly slide-free material it developed

into one giving greater slides than any others ; the last tests showed a value of $\pm$ 10 and $\pm$ 18 per cent. variation for the full and flex pressures respectively.

Artificial Slides.

The slides described above may be termed natural slides, inasmuch as no outside agency is required to produce them. They are not controllable either as to start or as to duration, and the plate-resistance brings them to a sudden stop ; although the slide shows the rise in pressure foreseen by the Author, it does not last long enough to show that a state of flux would ultimately bring the pressure back to normal. Each slide is an arrested state of flux, and there appears to be no means of preventing the plate from arresting it in this manner. If it is desired to cause a slide or state

Fig. 13.



of flux to last for some time, and to continue independently of the plate-resistance, it must be induced by some extraneous means capable of keeping up the state of flux.

An early attempt to produce this state of affairs was made in the following manner. During the filling of the test-box preparatory to a "flex" test (that is, a "sunk-wall" test) a rope was placed 2 inches behind the plate, as shown in *Fig. 13*, with a horizontal length lying parallel to the plate on every six-inch layer of fill, so that, on completion of the filling operation, a flat coil consisting of six turns of rope stood vertically behind the plate. This arrangement did not interfere with the flex test, the record of which showed the typical lines OA, OB and OC, the deflectometer at the end of the test standing at B. Upon the rope being pulled out, the plate immediately deflected outwards and the indicator rose towards A, but the state of flux did not last long enough to bring the pressure right up to normal.

The State of Flux.

It has been described on p. 107 how, at the end of a test, the soil is discharged into the lower store-chamber by opening the flap-valve d (*Fig. 4*). In the earlier tests this was done by loosening a cord holding up the flap. After a few tests it was noticed that, on discharging, the deflectometer moved violently and registered a sudden rise in pressure. Although at first sight it appeared peculiar that the pressure on the wall should increase when the fill was actually flowing away from it, it was realized afterwards that this was the state of flux, acting precisely in the manner anticipated. With this somewhat crude means of manipulating the flap-valve, it was not possible to measure exactly the rise engendered by the state of flux, and it was thought that the rush of grains towards the plate on their way to the discharge-opening might also have introduced the kinetic element, and thereby helped to push the plate forward. The method of operating the flap-valve was then altered to a lever arrangement, with which the valve could be moved as slowly as desired and stopped in any position of its travel, before actually opening to discharge the fill. This valve proved a most sensitive mechanism for inducing a state of flux, the least movement at any time being responded to by an immediate rise in pressure. So small is the movement required for starting and keeping up the state of flux that this can be done several times during a flex test, without the total travel of the valve opening up for discharge. The slightest movement of the valve apparently fluxes the whole vertical layer of soil nearest to the plate in a much more efficient manner than the rope described above.

As soon as the valve begins to move, the pressure rises, and continues to do so with the valve movement, until a certain value of pressure is reached, after which no further movement of the valve is of any avail. At this point the deflectometer remains constant, even if the valve is lowered far enough to open, and the fill is discharging. The rise thus registered is the lag, and no matter where the state of flux is applied during the test, whether during the initial stage or anywhere on the parallel stretch, the deflectometer rises to the check line and no farther. At every subsequent test carried out the state of flux has been tried, generally at the conclusion of the test, and all the trials gave the same result; in view of the unavoidable irregularities inherent in the material tested, the results are remarkably consistent.

Following the line of argument suggested in the Author's previous Paper [1] on this subject, it was decided to determine whether the

[1] *Ibid.*

state of flux would act in the same manner if the soil were subject to the opposite pressure-condition, namely, that of passive resistance, with condensed pressure, instead of the state of diluted pressure attendant upon the flex condition. In both cases the state of flux should bring the earth-pressure back to normal.

It is possible to produce conditions of passive earth-pressure in the testing machine by overloading the water drum during the balancing operations, and as the usual balancing load is about 17 inches of water and the drum can be filled to 30 inches, it is easy to give a considerable overload. When this is done, the plate exerts a pressure against the soil, corresponding to the amount of overload. With the thin plates the deflectometer does not register any inward movement, or compression of the soil, presumably because the plate is not stiff enough to let the surplus moment force back the filling. With the heavier plates a small, but distinct, compression is registered. When the state of flux is applied, as described above, during this overload or compression, the deflectometer invariably shows a drop in pressure, or inward movement of the plate, which corresponds exactly to the excess number of inches of water; that is, if the overload applied is 10 inches of water, the inward deflection is the same as that caused by this amount. In other words, the pressure in this case also reverts to the normal, or check-pressure. If, during the balancing at the beginning of a flex test, too much water has been let into the water drum, it is only necessary, on completion of filling, to move the flap valve slightly. The state of flux thereby induced at once sends the pressure back to normal, to the exact balancing value for the inches of water, the plate moving slightly inwards during fluxing. The same procedure may be followed if too little water has been let into the drum. The plate itself may not show any outward deflection when the deficiency in water is small, but the soil behind it may be in the state of diluted pressure, to the extent of the missing water-load; the application of the state of flux at once restores the pressure to normal, the plate moving outward to the extent corresponding to the missing "inches of water."

It is suggested that these experiments definitely prove the Author's assumption (6),[1] namely that a state of flux at any time nullifies the existing flex condition, whether positive or negative (that is, whether reduced or increased pressure), and brings about a reversion to normal pressure.

Stability of Flex Condition.

It would appear from the above that the flex condition is essentially unstable, whether in the form of diluted pressure behind a flexing

[1] *Ante*, p. 97.

wall, or as the condensed form known as passive earth-pressure. A state of flux will upset the flex condition as described.

The engineer is immediately confronted with the question as to whether the flex condition is sufficiently stable to be considered and utilized in everyday practice. Since the cause of its cessation is a state of flux, the question is actually as to whether a state of flux is likely to be induced or not. It should be mentioned that, although it was easy enough to produce a state of flux in the testing machine with absolutely dry sand, it was very difficult to do so when the sand became moist, even with the appliances to hand in the machine; the same observations would apply to soils with organic admixtures, or non-granular materials such as clay. Apart from this consideration, the ease or otherwise with which a state of flux may be induced depends to a great extent on the type of wall considered. In the case of the wall referred to above as a "sunk wall", it is difficult to imagine a state of affairs even remotely resembling that prevailing in the test-box during a state of flux, and it may be safely said that, both as regards diluted and condensed earth-pressures, advantage may be taken of the flex condition. Many wharves and other retaining walls are examples of this type of wall. The same may be said for walls of the "free" type, as long as a state of flux cannot occur. There are, however, instances, for example, bunker-walls, in which a hopper-opening may be situated near the side against which the pressure is acting. In such a case the opening of the hopper-valve may produce just such a state of flux as has been described above, and the wall may therefore be exposed to the full pressure of the material inside the bunker. In all such cases where a state of flux may possibly occur, no advantage should be taken either of the flex reduction to lighten the scantlings of the wall, or of the full passive resistance.

Balancing.

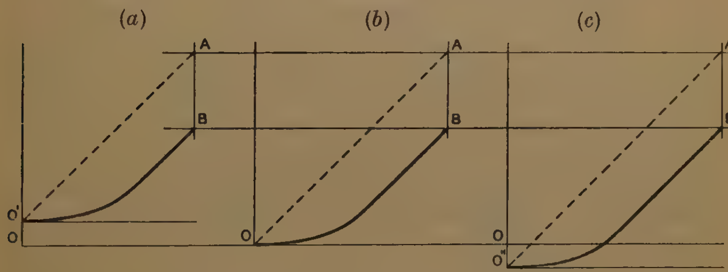
A few words must be said in regard to the method of balancing the earth-pressure by the moments at A and C (*Fig. 1*). As mentioned above, the plate is kept stationary during back-filling, either by means of straight-edges in front of it, or by applying the moments gradually as the filling goes on. In the earlier tests great trouble and care was taken over the correct balancing. In addition, the funicular for the two moments at A and C is not quite the same as that for the triangular earth-pressure diagram, and elaborate methods were adopted for correcting the slight error thereby introduced. As soon as it was realized, however, that by fluxing the soil a perfect state of equilibrium could be induced at any time, the necessity for painstaking balancing disappeared. Assuming the correct balancing

value for the normal earth-pressure to be 17 inches of water, then an error of, say 1 inch of water either way may not cause the plate to show any signs of over- or under-balancing, but immediately the fill is fluxed, the plate will deflect in or out, as the case may be, to the extent corresponding to exactly 1 inch of water. *Figs. 14* show the flex diagrams resulting from three tests with—

(A) too light balancing at 16 inches of water;
(B) correct balancing at 17 inches; and
(C) too heavy balancing at 18 inches.

OOO is the starting line, scribed by the indicator at the original position before the commencement of the test, with no moments at the supports and no filling. The flex diagram (*a*) represents the curve scribed by the deflectometer when the plate has been balanced at 16 inches of water. At the commencement of the

Figs. 14.

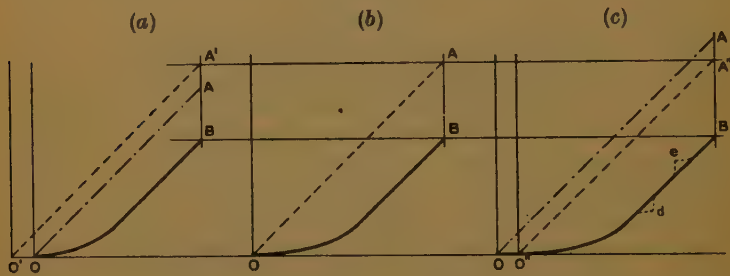


test the fill is fluxed, and the plate immediately moves outwards, sending the indicator up to O′, where OO′ is the deflection corresponding to 1 inch of water. The outflow of water is then started, together with the clockwork rotating the record drum, and at the end of the test, when the water-drum is empty, the flex-diagram O′B has been scribed, the check-curve for the constant, normal pressure being O′A. Upon fluxing the fill again, the pressure rises from B to A, which represents the amount of the lag. Diagram (*b*) represents the same test carried out with a plate balanced at 17 inches of water, which is the correct amount. The fill is fluxed at the beginning of the test, but nothing happens, and the indicator remains at O; upon starting the outflow as before, the indicator scribes the flex-curve OB, with lag BA, which disappears when fluxed again. In the third diagram (*c*), showing a test with 1 inch of water in excess of the amount required for balancing, the flux applied at the beginning of the test sends the plate inwards, and the indicator drops to O″, and the flex-curve is then O″B. In all three tests the actual deflection, from the starting line OOO to B, is the

same, as also the lag BA, and the total deflection corresponding to 17 inches of water. The three diagrams are exactly alike in shape, but of different lengths.

If, as in the earlier tests, no initial fluxing is applied to bring about an equilibrium, the diagrams in the three cases are not alike in shape. The flex-curves all end on the same level BBB as before (*Figs. 15*), and the fluxing after the test also brings the pressure up to the same level A′AA″, but whereas the curve in (*b*) is the same as in the previous tests, that in (*a*) rises more quickly and that in (*c*) more slowly. Similarly, if check-curves (chain-dotted) are plotted from the starting points O, the lag, while correct in (*b*), is too small in (*a*) and too great in (*c*); also, the final flux (at the end of the test) sends the pressure up to A′ in (*a*) which is higher than the check line OA, and in (*c*) sends it up to A″ which is lower than the check

Figs. 15.



line OA. The real check lines are O′A′ and O″A″, and the real deflections are exactly the same as in the previous example, but these circumstances were lost sight of in the earlier tests, and the seemingly variable results gave rise to a thorough investigation of the whole question of flux.

It will be seen that in all six tests the lag and the deflection are alike under whatever conditions the test is carried out, and in view of this it is not surprising that the results of repeated tests agree closely, although obtained at different times and in different circumstances. The method of finding the deflection and the lag is, in fact, practically fool-proof, and the flex-curve automatically tries to keep to the correct shape, even if external interference may produce irregularities in the line. If a small pebble becomes squeezed between the side of the box and the plate, and thus hinders the plate in deflecting, the small excrescence d (*Figs. 15* (*c*)) is caused on the flex-curve, but once the plate has passed the obstruction, or on removal of the latter, the indicator at once rises to the correct curve again. If a slide is induced, for instance, by fluxing for a moment, the peak e results,

but the deflectometer soon reverts to the proper flex-curve after the slide is over. Each of the natural slides is an attempt to divert the flex curve from its course, but the natural tendency is for it to go back again. In this sense, the flex condition may be said to be extremely stable, and, unless a state of flux should arise, it appears that only abnormal interference will prevent the soil from attaining or reverting to this condition.

Vibration.

During the tests attempts were made to ascertain the effect of vibration on the stability of the flex-condition. The difficulty of testing this on a small-scale model is that, relatively, the amplitude of the vibrations imparted to the apparatus generally exceeds very considerably that occurring with vibrations in the full-size wall. The shaking and hammering to which the test-box was subjected during these vibration-tests was more in the nature of an earthquake if applied proportionately to actual full-size conditions. It was found that vibration produced a certain rise in pressure, small in the case of thin plates, but more pronounced with the thicker plates. The latter seemed to be more sensitive to vibrations, both those artificially produced and those resulting from inevitable defects in the hoisting machinery and other causes beyond control. From the foregoing it will be clear that the probable reason for this rise is the introduction of a state of flux in the filling material, brought about by the vibration; it is not immediately clear, however, why a thick plate should be more sensitive to this kind of disturbance than a thin one. One explanation that suggests itself lies in a consideration of the grain-movements behind the plate during a state of flux; it is apparently the particles in the immediate neighbourhood of the plate whose movements determine the pressure-manifestations on the plate, and it is conceivable that, if such movements took place at a certain distance from the plate, say, behind the natural slope, no pressure-variation might ensue at all. Probably the whole of the unstable-pressure wedge of the fill is active and sensitive to the state of flux; in the apparatus used this state is always produced in close vicinity to the plate. For sand, the deflection of the thinnest plate is about $\frac{7}{8}$ inch for the fill in the flex condition, and the further deflection brought about by the state of flux is also about $\frac{7}{8}$ inch, as the lag is about 0·5. To bring about this deflection a volume of particles corresponding to the span of 36 inches and the deflection of $\frac{7}{8}$ inch must have been disturbed and set in motion by the state of flux. Compared to this, the thickest plate has a corresponding deflection of about $\frac{1}{16}$ inch, so that the volume to be set in motion by the state of flux, in order to produce the same increase from the

flex pressure to the full normal pressure, is only one-fourteenth of that required in the previous case. This agrees with the observations made during the flex tests, when it was found that, with the thick plate, only a very slight movement of the flap-valve was required to induce a state of flux sufficient to cancel the lag, whereas in the case of the thin plate a much greater travel of the valve had to be given.

Now, if a given amount of vibration is capable of shaking or moving a certain volume of particles into new positions, thereby producing a state resembling that of flux, it would follow from the above considerations that the effect, in terms of pressure, would be greater with the heavy plates and small deflections, and this assumption is borne out by the tests made to measure the effects of vibration. Whereas with the thinnest plates it was found practically impossible to decrease the lag more than 10 per cent., with the heaviest plate it was possible, by this means, to decrease the lag by more than 50 per cent. It must, however, be borne in mind, as mentioned above, that the vibrations employed in producing these results were abnormal, and would have no counterpart in real practice.

The method of filling in the soil behind the plate was not ideal inasmuch as a good deal of vibration was introduced. After a few skips have been filled into the top box, the discharged soil has to be raked forward against the plate, care being taken to keep the surface as level as possible. In this manner hoisting and raking operations are carried out alternately, and during the hoisting strong vibration is unavoidable. When the test is of the "sunk-wall" type, these vibrations do not matter, as the plate is kept stationary during filling through extraneous means, but if it is a "free-wall" test, the deflectometer shows a small rise in pressure every time the hoisting of the skip is causing vibrations. On the records it is possible, during the filling, to separate the periods of hoisting (vibration) from those of filling (raking), so that the gross deflection at the end of the filling may be split up into that due to filling proper and to that caused by intermittent vibration. Comparing the records from plate No. 1 with those from No. 4, it is found that the vibration contributions to the deflection are much heavier in the latter plate, and consequently the final flex pressure is also greater, or in other words, the lag is smaller in the heavier plate, although the full normal pressure, arrived at by fluxing the fill, is the same in both cases. The difference in the lag in this case is, in the Author's opinion, mostly due to the effects of vibration, and would not have occurred if it had been possible to fill the apparatus by a method entirely free from vibration.

Although, in tests of the "sunk-wall" type, vibrations should not affect the plate in this manner, since it is held during the filling

operations, it is nevertheless the case that the lag seems to decrease conversely with the thickness of the plate. During flexure there are no forces tending to produce vibration ; the unloading of the water is quite gradual, and care was taken that the apparatus was not disturbed or shaken while the plate deflected. Having therefore eliminated, as far as possible, the vibration factor, it remains to be explained why the lag should be smaller with the thicker walls.

Influence of Wall-Thickness.

The suggestion offered to account for this fact is the same as in the case of vibrations : the heavier plates are more sensitive to flux. While the plate is bending outwards the soil is set in the flex condition, which may be visualized as a state of looser packing, extending over a large volume in the case of large deflections (possibly right up to the surface-level), and over a smaller volume with the heavier plates. The grains actually in contact with the plate must necessarily be moving during the deflection, and these grain-movements may constitute the beginnings of a flux ; if they are not kept within control they may develop into a state of flux, such as may happen during a slide. The plate-resistance keeps these grain-movements from taking the form of a flux, sometimes with temporary interruptions as slides, sometimes without. In other words, a subdued flux is constantly occurring, or on the point of occurring, amongst the grains immediately behind the plate, and this flux, or tendency to flux, is acting adversely to the flex condition, inasmuch as it tends to raise the pressure which the flex condition has lowered. The question as to how far this plate-flux will succeed in interfering with the flex condition will presumably depend on the relative volumes in the one or the other condition. With thin plates and large deflections, the volume of particles on the move immediately behind the plate is insignificant compared to the large volume set in the flex condition, whereas with small deflections there may be a greater predominance of these flux movements.

Whether this is the real explanation of the smaller lag in the heavier plates, the Author is not prepared to say, and other opinions on this particular aspect would be welcome. In any case it means that, whereas theoretically the lag is constant for all deflections and wall-thicknesses, in practice the heavier walls show a smaller lag. This would agree to a certain extent with the suggestion put forward in the Author's previous Paper,[1] namely, that the reduction in earth-pressure on a flexible wall decreases with the wall-thickness, although for a reason entirely different from that advanced at the

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[1] *Ibid.*

time.[1] It was then thought that the reduction was a function of the deflection, increasing with the latter and disappearing when the wall became too stiff to deflect sensibly. The tests submitted in the present Paper have proved definitely that the lag, once having reached its full value, is constant for any deflection and independent of it. Any variation in the lag must be due to other causes, and it is suggested that the greater responsiveness to flux displayed by the heavier walls may be one of these causes.

Reactions.

During the earlier tests it was noticed that the heavier plates showed greater deflections, other conditions being the same. On investigating the reason for this, it was found that, as the top and bottom supports (A and C, *Fig. 1*) were not absolutely immovable, the slight elastic "give" accounted for this discrepancy. With the thinner plates the deflection is so great that this yielding of the supports is almost negligible in comparison, but with the heavy plates and small deflections, the proportion of the "give" to the deflection becomes a factor of some importance. The actual elastic yield of the supports amounts to approximately 0·0009 inch per inch of water, and as the magnification used for the heavy plates in some cases amounted to over 100, the yield for, say 17 inches of water, showed as more than 1 inch, and could thus be measured easily. In all the later tests the deflection due to the yield of the supports was measured separately and taken into account; this necessitated a good many more readings, but it also afforded a very simple means of demonstrating the correctness of assumption No. 4,* namely that the flex condition does not apply to the reactions. After having ascertained the magnitude of the yield of the supports for 1 inch of water, it is possible to measure the earth-pressure by measuring the yield produced by it. This is not a very accurate method of determining earth-pressures, as the elastic "give" is so very small, but reasonable agreement was found between the earth-pressure measured by the yield and that measured by balancing, as described previously. The yield was measured under exactly the same conditions as those obtaining under the proper earth-pressure tests, both with "sunk" and "free" walls, and the interesting point is

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[1] In "Concrete Structures in Marine Work," p. 27, Fig. 20, a set of reduction curves for reinforced concrete walls is given, in which the reduction is shown as diminishing with increasing wall thickness, so that the maximum reduction is assumed to be present with a minimum thickness of one-fiftieth of the span, and decreasing to zero with a wall thickness equal to the span. The maximum reduction is here based upon the reduction factor being of the order of $2f/(1+f^{\frac{3}{2}})$.

* *Ante*, p. 97.

that the fluxing operation, which increases the plate-pressure to nearly double, made not the slightest difference to the reaction pressure. This latter was the same both before and after fluxing, and corresponded to the balancing value, which is the normal earth-pressure. The obvious conclusion is that the reactions suffer no diminution by what has been described as the flex condition, and that the reduction caused thereby is local only, and merely a redistribution of pressure on the flexing wall.

Some authorities on this question assert that a reduction in the reactions does take place, so that, for instance, the anchorage of a sheet-wall may be lightened under these conditions, but it seems difficult, in view of the test-results mentioned, to uphold this contention.

The saving, from the point of view of the engineer, lies in the reduction of the bending-moment on the wall, and not in any effect on the anchorages, whether at the top in the form of tie-backs, or at the bottom as passive earth-pressure. The reduction of wall-scantlings, however, is well worth considering. Taking, for instance, a case with a reduction to one-half, and stiffening this to, say five-eighths to allow for contingencies such as strong vibration, this would mean a saving of 20 per cent. in the scantlings.

Test-Results.

The particulars of the plates used in the tests were as follows :—

Plate No.	Thickness: inches.	Moment of resistance: inches^3.	Moment of inertia: inches^4 ($\times 10^6$).	Deflection per inch of water: inches.	Modulus of elasticity: millions of pounds per square inch.	Magnification generally used.
1	0·109	0·0713	3,885	0·10500	33·1	$9\frac{1}{2}$
2	0·125	0·0937	5,859	0·06520	35·5	$12\frac{3}{4}$
3	0·175	0·1837	16,078	0·02340	35·8	36
4	0·255	0.3901	49,744	0·00784	34·7	82–118

A summary is now given of the measured values of the total or normal earth-pressure, and of the lag, for different soils, with the four plates. The difference between Rankine's value of the earth-pressure and the measured balancing value was to be expected, on account of the friction on the sides of the box.

Sand.

This material consisted of washed Thames sand, containing a large amount of smalls down to dust. Its grain-analysis is given below, and it is considered that this represents a fair example of the sand generally used in practice.

GRAIN-ANALYSIS.

10	per cent.	between	$\frac{1}{4}$	inch and	$\frac{1}{8}$	inch.
9	,,	,,	,,	$\frac{1}{8}$	,,	$\frac{1}{10}$	,,
16	,,	,,	,,	$\frac{1}{10}$	,,	$\frac{1}{20}$	,,
15	,,	,,	,,	$\frac{1}{20}$	,,	$\frac{1}{30}$	,,
10	,,	,,	,,	$\frac{1}{30}$	,,	$\frac{1}{40}$	,,
10	,,	,,	,,	$\frac{1}{40}$	,,	$\frac{1}{50}$	,,
17	,,	,,	,,	$\frac{1}{50}$	,,	$\frac{1}{76}$	,,
8	,,	,,	,,	$\frac{1}{76}$	,,	$\frac{1}{100}$	,,
3	,,	,,	,,	$\frac{1}{100}$	,,	$\frac{1}{180}$	,,
2	,,	,,	less than $\frac{1}{180}$ inch.				

The sand was dried for several months before the tests, and contained no moisture during testing; its weight per cubic foot was found to be 108 pounds. This is the loose weight; if the sand were shaken or consolidated, the weight might increase by 10 per cent. or more. The natural slope varied with the circumstances in which it was measured; the sand left in the test-box after emptying into the store space below showed a slope of 35 degrees for the inert material, which would correspond to a value of $f = 0{\cdot}271$. This sand was the first material to be used in the tests and a great number of records have been obtained, of which *Figs. 8, 9, 10* and *11* (p. 115) show typical examples taken with plates Nos. 1, 2, 3 and 4 respectively. These figures are reproductions of the actual records, but they have been altered in scale to facilitate comparison with one another; the slides were very distinct with plates 1, 2 and 3, but never occurred with plate 4. Their length corresponded to approximately $\frac{7}{8}$ inch of water, and thus represented about $\pm$ $2\frac{1}{2}$ per cent. and $\pm$ 5 per cent. of the total and of the flex pressure respectively. The characteristics of this material remained constant throughout the period during which it was tested, the first tests being in 1930 and the last in 1935; one of the difficulties in these, as in nearly all the tests, is to determine the natural slope, and the figure of 35 degrees is only an average value.

SAND TESTS NOS. 262–319, 462–520 AND 597–602.

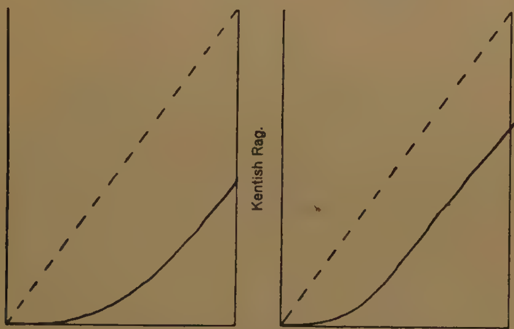
Plate No.	Total earth-pressure: inches of water.	Earth-pressure: pounds.	Rankine's value: pounds.	Lag l: inches of water.	l: pounds.	l in terms of total earth-pressure.	Reduction factor r in terms of earth-pressure.
1	16·70	369	400	9·00	199	0·54	0·46
2	16·60	367	400	8·92	197	0·54	0·46
3	16·62	368	400	8·54	189	0·51	0·49
4	16·70	369	400	8·16	180	0·49	0·51

In all tests, 1 inch of water represents a pressure of 22·08 pounds on the plate, as described on p. 110.

Kentish Rag-Dust.

This material is the siftings from Kentish rag-stone, and is a whitish, floury dust which will stand at almost any angle. Its natural slope is somewhat indeterminate, but the average from a number of tests is 37 degrees, its loose weight being about 90 pounds per cubic foot; no slides were shown with any plate. This material behaved less like a granular filling than any of the others, and the test-results vary within wider margins. Although the lag for plate No. 1 is practically the same as with sand, it decreases more with the plate thickness than in the case of the other filling materials. *Fig. 16* shows one of the records with plate No. 1, and *Fig. 17* one with plate No. 4.

Fig. 16. *Fig. 17.*



KENTISH RAG TESTS, NOS. 322–370.

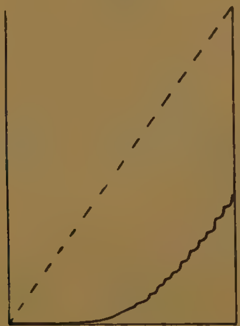
Plate No.	Total earth-pressure: inches of water.	Earth-pressure: pounds.	Rankine's value: pounds.	Lag l: inches of water.	l: pounds.	l in terms of total earth-pressure.	Reduction factor r in terms of total earth-pressure.
1	13·0	287	309	7·15	158	0·55	0·45
2	12·63	280	309	6·63	146	0·52	0·48
3	12·13	268	309	5·11	113	0·42	0·58
4	12·37	273	309	4·30	95	0·35	0·65

Granite Chips.

The particles vary in size, from $\frac{1}{16}$ inch to $\frac{3}{8}$ inch being the greatest dimension; they are flat in shape and have a thickness of from $\frac{1}{32}$ inch to $\frac{1}{8}$ inch. Their surface is rough and they stand up to a slope of

40 degrees, the weight being 87 pounds per cubic foot. Only plates Nos. 1, 3 and 4 were tested, and slides showed with No. 1 but not with Nos. 3 and 4. The average length of a slide corresponded to about $\frac{5}{16}$ inch of water, and thus represents only $\pm$ 1·5 per cent. and $\pm$ 3·5 per cent. of the total and of the flex pressure respectively. *Figs. 18* and *19* show records of plates Nos. 1 and 4.

Fig. 18.



Granite Chips.

Fig. 19.

GRANITE CHIPS TESTS, NOS. 530–584.

Plate No.	Total earth-pressure: inches of water.	Earth-pressure: pounds.	Rankine's value: pounds.	Lag l: inches of water.	l: pounds.	l in terms of total earth-pressure.	Reduction factor r in terms of total earth-pressure.
1	11·25	246	258	6·50	143	0·578	0·422
3	10·30	226	258	5·37	118	0·522	0·478
4	10·62	233	258	5·22	115	0·492	0·508

Pea Gravel.

The particles are fairly uniform in shape, well rounded and varying in size from $\frac{3}{16}$ inch to $\frac{3}{8}$ inch. Their surface was fairly rough at the beginning of the tests, but as the pebbles became worn by the many chargings and dischargings into and from the test-box, the roughness wore off and their surface became quite smooth. This influenced their behaviour somewhat, and whereas originally they showed slides on the same three plates as in the sand tests, the former disappeared gradually, and the last tests presented a continuous deflection-line with all plates. The characteristics of the gravel also underwent a slight change, the slope flattening from 31 to 30 degrees and the weight increasing from 89 to 90 pounds per cubic foot. The slides in the first tests were small but quite distinct; *Fig. 20* shows a test with plate No. 1, and the slide measured about

PEA GRAVEL TESTS.

Plate No.	Total earth-pressure: inches of water. Test Nos.		Earth-pressure: pounds. Test Nos.		Rankine's value: pounds. Test Nos.		Lag l: inches of water. Test Nos.		l: pounds. Test Nos.		l in terms of total earth-pressure. Test Nos.		Reduction factor r in terms of total earth-pressure. Test Nos.	
	153–215	418–461	153–215	418–461	153–215	418–461	153–215	418–461	153–215	418–461	153–215	418–461	153–215	418–461
1	14·37	15·20	317	336	383	400	7·33	7·62	158	168	0·51	0·50	0·49	0·50
2	14·00	15·35	310	340	383	400	7·25	7·70	157	170	0·51	0·50	0·49	0·50
3	14·15	16·10	312	355	383	400	6·35	7·13	140	156	0·45	0·44	0·55	0·56
4	15·55	17·25	344	381	383	400	6·90	6·80	152	151	0·44	0·40	0·56	0·60

$\frac{3}{8}$ inch of water, which corresponds to $\pm$ 1·3 and $\pm$ 2·6 per cent. of the total and the flex pressures respectively. *Fig. 21* shows one of the latest records on the same plate, and is without slides.

Fig. 20.

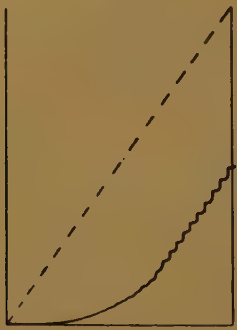
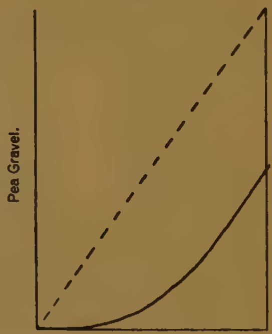


Fig. 21.

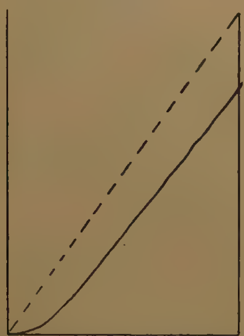


Linseed.

The sole reason for obtaining this material was the fact that the samples which were examined before buying showed a very high degree of liquidity, the natural slope being only 24 degrees and the weight 40 pounds per cubic foot. The material is very lively and flows very easily, the individual seeds having a highly polished surface and all having practically the same size and form. As a granular material it is perfect. The seeds are about $\frac{3}{16}$ inch long by $\frac{3}{32}$ inch wide and $\frac{1}{32}$ inch thick, and approximately oval in all three sections. As has been mentioned before, the properties of the linseed altered very quickly, so that, in the time elapsing during the tests, namely from the 29th October to the 20th November, 1932, the material underwent a complete change in regard to slope, weight, slides and lag. The seeds seemed to be very sensitive to moisture, and in damp weather appeared to exude a sticky substance, which had the effect of preventing the easy moving of the particles, both by the slight adhesion of the grains, and also by collecting every particle of dust that came in contact with them. This discharge of gluey material was very noticeable when the seeds were moistened accidentally; a small quantity which was spilt on the ground, and was then exposed to moisture, became congealed into a solid lump. Two circumstances combined to effect the rapid change in properties, namely, that the month of November, 1932, was rather damp, and that the test-shed adjoined a coke-screening plant, the dust from which pervaded the whole of the surroundings. Although to the naked eye there was no change in the appearance of the individual seeds, the microscope showed a coating of fine coke-dust which was responsible for the

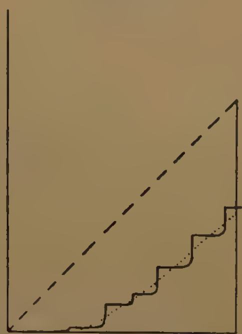
great change in properties as a fill material. The first test was on a "free" wall to ascertain the total deflection; this was to be used for balancing purposes in the next test, which was on a "sunk" wall. The first reading gave a total of 8·375 inches of water, equivalent to 180 pounds, but in the comparatively short space of time elapsing between this and the next test, this value dropped to 7·4 inches of water, or 163 pounds, and showed a lag of 1·7 inches of water; in terms of the total, l was 0·23, and the reduction factor r was 0·77. This was on the thinnest plate, No. 1, and the deflection line was quite smooth and without slides, as in *Fig. 22.* Only three tests could be performed that day, the third giving a total of 7·0 inches of water or 154 pounds, with a lag of 0·25 and reduction factor of 0·75. The slope had by then already stiffened to 26 degrees. As soon as it

Fig. 22.



Linseed.

Fig. 23.



was realized that the characteristics of the material were still changing, the tests were rushed through, and during the dozen tests on this plate the total pressure gradually diminished to 6·375 inches of water, or 140 pounds, with a lag of 0·275 and a reduction factor of 0·725. The slope by that time had increased to nearly 30 degrees and the weight decreased to just under 38 pounds per cubic foot, and small slides were beginning to show. During the remaining tests the properties of the material continued to alter in the same direction, though not so quickly as at first, until, 3 weeks later, the slope was 33 degrees and the weight 36 pounds per cubic foot. Slides became very pronounced, even with the thickest plate, this being the only material that showed slides with this plate, and the last tests, which were carried out with plate No. 1 to obtain a comparison with the first tests, gave slides of 1 inch of water, a very high value compared to the other materials. The total, or balancing, value was then measured as 5·375 inches of water, corresponding to a pressure of 118 pounds, with a lag of 0·46 and a value for r of 0·54; this record is shown in *Fig. 23* to compare with *Fig. 22.*

Although, as a test-material in the ordinary sense, the linseed was a failure, on the other hand the changes in its properties and behaviour threw a very interesting light on the interaction of the factors that are the subject of this investigation, and for that reason the time and money spent on it are not considered wasted.

LINSEED TESTS, NOS. 216–261.

Plate No.	Total earth-pressure: inches of water.	Earth-pressure: pounds.	Rankine's value: pounds.	Lag l: inches of water.	l in terms of total earth-pressure.	Reduction factor r in terms of total earth-pressure.	Remarks.
1	8·375	180	224	—	—	—	First test, filling.
	7·400	163	206	1·70	0·23	0·77	First test, flex.
	6·375	140	173	1·75	0·275	0·725	Last test, flex.
2	6·0	132	163	1·97	0·33	0·67	Average values.
3	6·8	149	151	2·35	0·35	0·65	Average values.
4	6·6	145	145	2·50	0·38	0·62	Average values.
1	5·375	118	143	2·50	0·46	0·54	Final test.

In the Table on p. 137, the figures from the five materials have been arranged in order to afford a comparative survey of the test results.

CONCLUSIONS.

The experiments may be said to have furnished the following facts in regard to the granular soils tested :—

(*a*) The bending moment on a wall flexing between its supports is definitely reduced. The reduction is purely a phenomenon due to the presence of double friction, and it is constant for any deflection.

(*b*) The reduction is a re-distribution only of pressure on the wall, and there is no diminution in the reactions, which are equal to the total earth-pressure.

(*c*) The reduction is a function of the friction-angle, which is the natural slope in granular materials, and the reduction-factor r is approximately equal to $2f/(1+f^2)$. The reduction is less pronounced with heavy than with thin walls, as described in (*d*) below, and this value of r would be applicable to thin walls only; for reinforced concrete walls a thickness of, say 1/50 of the span, would be a "thin wall." The increase of the reduction-factor with the wall-thickness has been taken into account in sets of reduction-curves previously published by the Author.

As the reduction is due to the double friction, it is reasonable to assume that it must also be present in soils with combined friction and cohesion. The expression for the earth-pressure in these soils contains two factors, namely, the friction element and the cohesion

Material.	Earth-pressure totals measured in inches of water. Lags l and reduction factors r measured in terms of earth-pressures.												Natural slope: degrees.	Liquidity factor, f.	Weight: pounds per cubic foot.	Rankine's value expressed in inches of water.	Slides measured in inches of water.
	Plate No. 1.			Plate No. 2.			Plate No. 3.			Plate No. 4.							
	Earth-pressure.	l.	r.	Earth-pressure.	l.	r.	Earth-pressure.	l.	r.	Earth-pressure.	l.	r.					
Granite chips .	11·25	0·58	0·42	—	—	—	10·30	0·52	0·48	10·62	0·49	0·51	40	0·22	87	11·8	Plate 1 only, $\frac{5}{16}''$.
Kentish rag .	13·00	0·55	0·45	12·00	0·52	0·48	12·13	0·42	0·58	12·37	0·35	0·65	37	0·25	90	13·9	Non-slip.
Sand .	16·45	0·55	0·45	16·10	0·54	0·46	16·81	0·51	0·49	17·00	0·47	0·53	35½	0·27	108	18·3	Plates 1, 2 & 3, $\frac{7}{8}''$.
Pea-gravel	14·37	0·51	0·49	14·00	0·51	0·49	14·15	0·45	0·55	15·50	0·44	0·56	31	0·32	89	17·3	Plates 1, 2 & 3, $\frac{3}{8}''$.
Do. later.	15·20	0·50	0·50	15·35	0·50	0·50	16·10	0·44	0·56	17·25	0·39	0·61	30	0·33	90	18·3	Non-slip.
Linseed .	8·375	—	—	—	—	—	—	—	—	—	—	—	24¼	0·418	40	10·2	Non-slip.
,,	7·40	0·23	0·77	—	—	—	—	—	—	—	—	—	26	0·392	39	9·4	Non-slip.
,,	—	—	—	6·00	0·33	0·67	—	—	—	—	—	—	30	0·333	37	7·4	Small slips.
,,	—	—	—	—	—	—	6·80	0·35	0·65	—	—	—	32	0·306	36½	6·83	Slides increasing.
,,	—	—	—	—	—	—	—	—	—	6·60	0·38	0·62	32½	0·3	36	6·6	Slides increasing.
,,	5·375	0·46	0·54	—	—	—	—	—	—	—	—	—	33	0·295	36	6·5	Slides 1″.

element, which reduces the former; the friction element would be subject to the same reduction as in a purely granular material. Even in a purely cohesive soil, if such a filling material existed, the same principle of double cohesion would seem to obtain and would produce a corresponding reduction in the bending moment.

(*d*) If a state of flux can occur in the soil, the condition of reduced pressure is nullified, and this also applies to a state of increased pressure, or passive earth-pressure. If the flux is of sufficient duration, the pressure in both cases reverts to normal.

Thick walls are more sensitive to a state of flux than thin walls, and it is thought that this may be the reason for the reduction in pressure being less pronounced with heavy walls. For the same reason thick walls are more sensitive to vibration, which produces a state similar to that of flux.

(*e*) A state of flux cannot occur in cohesive materials, but only in purely granular soils. Even in the latter case a state of flux is rarely possible in practice, except in bunkers having a certain type of outlet.

(*f*) Slides may take place in granular materials during flexure, but, although a small rise in pressure occurs during each slide, the effect may be considered to be negligible.

In conclusion, the Author would like to express his appreciation and thanks to Mr. C. M. Croft, M. Inst. C.E., Engineer and Manager of the Wandsworth and District Gas Co., for his courtesy in placing at the Author's disposal the facilities for carrying out the tests on the company's premises.

The Paper is accompanied by seven sheets of diagrams and one photograph, from which the Figures and half-tone page-plate in the text have been prepared, and by the following Appendix.

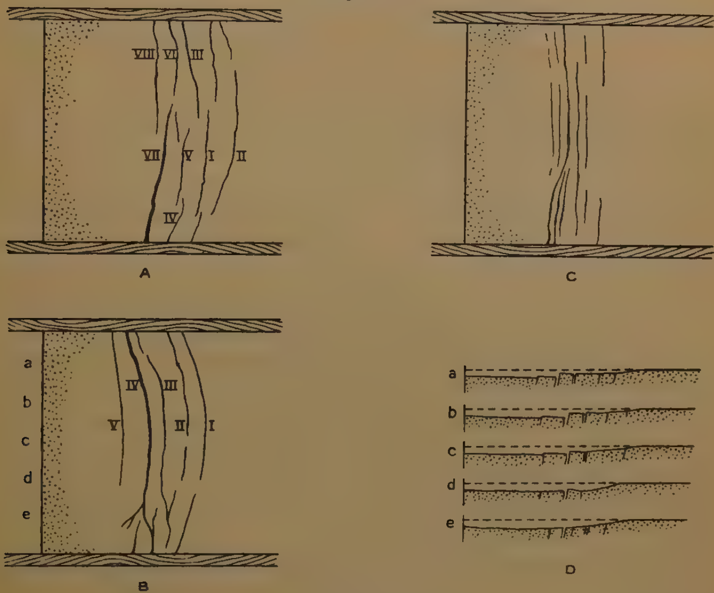
APPENDIX

Rupture Planes.

As a matter of interest, a few notes are appended on the observations taken, as a side issue, when testing two of the materials that showed definite rupture planes, namely, Kentish rag-dust and moist sand. The surface of the other materials never gave any clear indication of rupture planes; nor was this to be expected with dry and truly granular material, where a cleft cannot form and last for an appreciable time. A sinking of the soil-level behind the plate, extending back roughly half the plate-span, was observed when very large plate-deflections took place with these two materials, while the level behind this remained at its original height; the difference between the two levels was marked by a fairly distinct border

line. In Kentish rag and moist sand, the fissures formed and stayed during the deflection tests with "sunk" walls, some remaining small and showing only as a line, others growing in width up to $\frac{5}{8}$ inch across, according to the deflection of the plate. Observation was kept during the deflection so as to ascertain the position, width and order of formation of these rupture lines; both form and position were naturally irregular, but it seemed to be the rule that the first cleavages appeared farther back, up to about 30 inches behind the plate, followed by others nearer the plate. While the

Figs. 24.



first lines never developed to more than about $\frac{1}{32}$ inch or $\frac{1}{16}$ inch in width, the later and foremost fissures appeared to be the heads of the main rupture planes, and these extended to widths up to $\frac{5}{8}$ inch. The widest clefts appeared about 18 inches behind the plate; after they had started all further movement seemed to be confined to the plane indicated by them. *Figs. 24* (A) and (B) show, to scale, plans of the sand surface in the test-box after the formation of these rupture lines, which are marked with numbers in the order in which they appeared; *Figs. 24* (C) is a plan of the Kentish rag lines; while *Figs. 24* (D) shows a vertical section through sand, from which the change in levels will be apparent.

The Council invite written communications on the foregoing Paper, which should be submitted not later than three months after the date of publication. Provided that there is a satisfactory response to this invitation it is proposed, in due course, to consider the question of publishing such communications, together with the Author's reply.

SPECIAL LECTURE ON

"Surveying from Air Photographs."

BY BREVET-MAJOR MARTIN HOTINE, R.E.

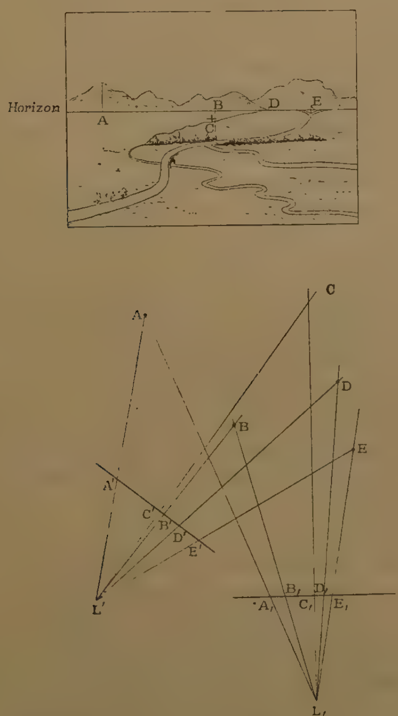
THE first extensive use of ground photography, as an aid to surveying, was made in the Canadian Rockies towards the end of the last century, the main object being to secure as much information of inaccessible country as possible during a short field season. Levelled photographs were exposed in measured directions from known positions, and used later to derive the positions and heights of features common to two photographs by means of graphical intersection. This method is illustrated in *Figs. 1.*

A difficulty which frequently arose in this method was to obtain a positive identification of a feature which appeared on two widely dissimilar views. In the case illustrated, there would be no difficulty in locating a definite object, such as a church spire, on two photographs, and thus intersecting it with certainty, but it is easily seen that no such assurance could be secured if the church spire were replaced by an indefinite ridge or peak. An obvious solution of this difficulty was to expose the two photographs from the ends of a short base, so that the two views should not be very different, but the resulting narrow graphical intersections then become too inaccurate. Consequently, the next step was to place the two short-base photographs in a measuring stereoscope, which completely solved the difficulty of identification, and to obtain distances by a micrometric measurement of stereoscopic parallax instead of by graphical intersection. For this solution of the problem we are indebted to another British Dominion, although later, an independent result was obtained in Germany. The method was first applied in practice, with an instrument of his own design, by Fourcade, a South African forestry officer, who found it necessary to make his own maps before he could do any useful forestry work.

The next stage in development was provided by Thompson, a British Army Officer, who succeeded in avoiding a certain amount of computation by adding to the stereoscopic measuring machine a system of plotting-levers and a drum, from which he could read

off distances directly. His machine was immediately improved by Zeiss, and a succession of plotting machines of ever-increasing complexity have since been produced by various Continental firms. Perhaps the most successful is the Wild Autograph shown in Fig. 2, Plate 1, which, unlike its humble forbears, can deal with photographs exposed on any orientation or tilt, whether on the ground or from the air, and, once the pair of photographs is correctly set, can plot

Figs. 1.



a continuously-drawn plan or contour from them without any computation of particular positions or heights. It is important, however, to note that all these machines were developed as an extension of the problem of ground photography, where the position, tilt and orientation of the camera could all be measured accurately at the time of exposure, for subsequent setting in the machine. No such external data can be provided by direct measurement in an aircraft subject to random accelerations, and this fundamental difference between ground and air photography ought properly to involve an entirely different approach to the problem of utilizing the

air photograph. Nevertheless, this point has been consistently missed on the Continent. Machines which were doubtless suitable for ground photography have been crudely adapted to take air photographs, without considering that the problem of setting the latter is fundamental, resulting in undue complication and cost.

It will be apparent that ground photographic survey is applicable mainly to mountainous country, where a sufficient view can be obtained, to make the method economically worth while as compared with non-photographic methods. In flat enclosed country, it is clearly useless to fix a pair of camera-stations, to expose two carefully oriented and levelled photographs, and subsequently to labour over them in the office, solely in order to fix the position of the one building which may appear on both. This limitation in view from ground-stations has been completely overcome by vertical photography from the air. The fact that a vertical air photograph is usually in itself an approximate plan, which a ground photograph can never be, is a further reason for treating it *de novo*, without any bias from previous ground methods.

Apart from the utilization of air photographs for revising old maps during the World War, it is necessary to return to Canada for the first extensively applied method of using air photography for mapping. Oblique photographs were exposed to include an image of the horizon, from which the tilt of the camera could be deduced. This, together with an altimeter measurement of height or a measured base on the ground, enabled a perspective grid to be fitted over the photograph as shown in Fig. 3, Plate 1. Detail could then be transferred "square" by "square" to the corresponding square of the map-grid.

This method, being based on the principles of plane perspective, is only applicable to reasonably flat country and to clear-cut detail such as may clearly be seen in the background of oblique photographs, but it has been responsible for a large volume of very useful work in the Laurentian lake and forest country, for which it was devised. In common with the first applications of ground photography in the Canadian Rockies, it is due to Deville, who, although a surveyor of international scientific reputation, never seems to have made the mistake of confusing a practical map with a cross between a work of art and a geometrical figure.

It will be seen that an oblique photograph shares to some extent the restriction in view so apparent in ground photography; it is, in fact, half-way between the ground photograph and the very detailed vertical air photograph. Important detail may easily be hidden by trees, while a low hill in the background of an oblique photograph may blot out several square miles of country. For

this reason, it has become usual of recent years to concentrate on the approximate plan view of the vertical photograph, and it is mainly in this direction that developments in this country and in America have taken place. In this country we suffer from the disability of being able to secure accurate, if, unfortunately, not always up-to-date, maps on almost any scale between sixteen miles to the inch and twenty-five inches to the mile for the modest sum of six shillings and eightpence or less. There has accordingly been little incentive to develop new methods of surveying in the Universities, and work on air survey has been largely confined to the Army and Royal Air Force, and to certain civilian firms operating mostly abroad. Nevertheless, there are considerable unmapped areas in the British Empire, if not in the British Isles, and there can be no doubt that development of our Colonies ought to rest on rapid, accurate and complete topographic information to a far greater extent than is usually the case at present.

An untilted vertical photograph of flat country would actually be a true plan, and a succession of such photographs taken from the same height would have a uniform scale. In cases where these conditions are nearly fulfilled, it is quite possible therefore to join the photographs together to form a composite photographic plan or mosaic. Variations in scale are, however, introduced by tilts of the camera and by differences in ground-heights, for the simple enough reason that certain objects are thus brought relatively nearer to the camera. Anyone who has been caught in a lounging attitude by a photographer at a picnic will appreciate this. In such cases, any attempt to join the actual photographs together must result in certain areas being omitted or duplicated; a complete representation of the ground is not secured, even if an accurate plan is not required, and it becomes necessary to produce a line map. Where tilts and variations in height are not excessive, we may assume that angles subtended at the centre of the photograph are true horizontal angles, and we may then construct a line map by resection of the photographs and intersection of the detail appearing on them. Several photographs may be strung together on this assumption and the resulting plan fitted to an open framework of positions fixed on the ground. A succession of three photographs marked up in this manner with radial lines for graphical assembly and plotting, is shown in Figs. 4, Plate 1.

The measurement of heights is very considerably simplified if the photographs are untilted and exposed from the same altitude, in which case no more complicated instrument is required than the measuring stereoscope shown in Fig. 5, Plate 1. For small departures from these conditions, a series of simple corrections can

be applied which require no more data than an accurate measurement of the difference in flying altitude, by means of a sensitive statoscope or a differential altimeter.

The whole future of the method seems to me to depend on providing greater accuracy in the photographic data rather than on greater complexity in dealing with faulty photographs, on the virtual elimination of tilt by gyroscopic stabilization of the aircraft controls, on the provision of accurate measures of altitude, and on the use of undistorted photographic materials. All these improvements are available, but they have not yet been fully utilized. When they are utilized, I venture to predict that it will be possible to produce entirely adequate large- or medium-scale fully-contoured topographic maps of any type of country by these very simple methods with no more ground-surveyed positions and heights than an open triangulation.

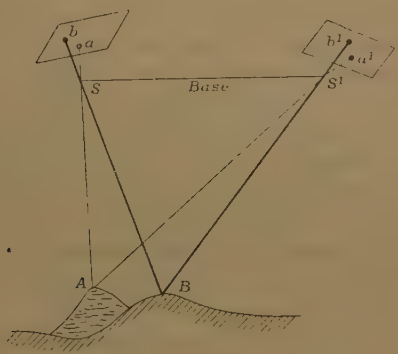
The same methods may be used for large-scale plans if correspondingly large-scale photographs are used and a somewhat closer ground control is made available, but it is unlikely that the air photograph will ever be used for providing levels of the accuracy usually associated with large-scale engineering plans. The fraction of a foot required, when translated into the stereoscopic parallax used for its measurement, is so small a quantity as to be confused with the grain-size of the photographic emulsion. The representation of ground-forms by contours, which are necessarily subject to errors of generalization and of faulty registration in colour-printing, can be provided by one method or another to a sufficient degree of accuracy, but spot-levels to the degree of accuracy usually associated with spot-levels cannot be provided. In a few words, the air photograph can replace the clinometer, the plane-table and to a large extent the chain, but it cannot replace the spirit-level and the theodolite.

For certain classes of work, where a sufficient ground-control cannot be provided or where greater accuracy, particularly in contouring, is required, some more rigorous instrumental method is desirable. For this purpose, a machine known as the Fourcade Stereogoniometer is being developed. This instrument differs radically from the Continental machines in that it is designed primarily to utilize air-photographs on correct principles of stereoscopic perspective. As a result it gains in simplicity and ease of setting and it becomes possible to use the machine simply for establishing a framework of accurate control-positions and heights. The next model to be constructed by Messrs. Barr and Stroud will be provided with an automatic plotting mechanism for the continuous drawing of plans and heights. The first model was con-

structed solely as a control instrument, comparable to a double theodolite.

The basic principle of this Fourcade instrument is very simple indeed. If two photographs are exposed from the ends of an air base-line SS^1, as in *Fig. 6*, an object A on the ground gives rise to photographic images a, a^1. It is obvious that the lines ASa, AS^1a^1 must lie in a common plane, for the reason that they intersect in a point A. Consequently the images a, a^1 are in a plane containing the base SS^1, and the same applies to any other pair of images b, b^1. Conversely, it may be proved that if the photographs are replaced in similar cameras or projectors to re-establish the perspective conditions, and the two perspective units are so arranged that each

Fig. 6.



pair of corresponding images lies in a plane containing the base, then the photographs will occupy correct positions relative to each other and to the base, and a true-to-scale model of the landscape will be secured. Theoretically, it is sufficient to establish the correspondence of five pairs of images to secure this result, although more are used in practice, and the Fourcade instrument is specially designed to effect this setting rapidly and accurately. A whole strip or block of photographs may be strung together in this manner to form in effect a composite true-to-scale three-dimensional model of the landscape, the scale and level of which can be established from a few ground-surveyed positions and heights.

A large volume of topographic mapping, of a standard generally suitable for engineering reconnaissance, has already been produced by simple aerial photographic methods in odd corners of the Empire. The cost of a topographic air-survey, compared with normal plane-table methods, depends to a large extent on the size of the area and the type of country; a small isolated area cannot economically

be photographed from the air, and it is still cheaper to plane-table open hilly country containing easy communications. But this statement assumes that the total cost of photography has to be debited to mapping, whereas the photographs are of considerable value for a variety of other purposes connected with economic development. I shall refer to this point later. Apart from the question of cost, an air-survey can always be carried out far more quickly than a similar detail-survey on the ground, and it is this factor of speed which will eventually be realized to be its greatest engineering asset. Many major engineering projects have in the past been carried through on the basis of faulty topographic information, not because engineers fail to realize the value of topographic maps for the purpose, but because maps could not be provided by normal ground methods in time. There are many recorded examples, for instance, where faulty railway alignment, due to lack of sufficient topographic information over a large enough area, has resulted in excessive maintenance and subsequent realignment costs, which would have paid for a topographical survey of the whole country many times over. But maps could not at the time have been provided quickly enough. It is a lamentable fact in the history of almost every country that due provision for extensive topographic mapping is never made in time to be of most use, and if this lack of foresight persists, as it doubtless will, then the air photograph seems to be the only way of avoiding wasteful development of new country.

The provision of rapid and cheap topographic maps from air-photographs is likely to be considerably facilitated in future by the use of multi-lens cameras, covering a very much larger area of country in one exposure. Several models of this type already exist.

For the preparation of large-scale plans, the air photograph is almost invariably cheaper than ground methods if a sufficient area is in question, in spite of the fact that a greater proportion of the work, in comparison with smaller scales, remains to be done on the ground. It is, of course, only possible to plot such detail as can be seen on the photograph, and correctly interpreted with the help of a stereoscope; any further detail which may be required must be supplied on the ground.

Much the same conclusion applies to the revision of large-scale plans. If a considerable amount of extra detail has to be added, air photography of large blocks will pay both in time and money, although a certain amount of detail and examination must be added on the ground. It might well pay in the urban areas of this country, provided that arrangements are made for economic photography

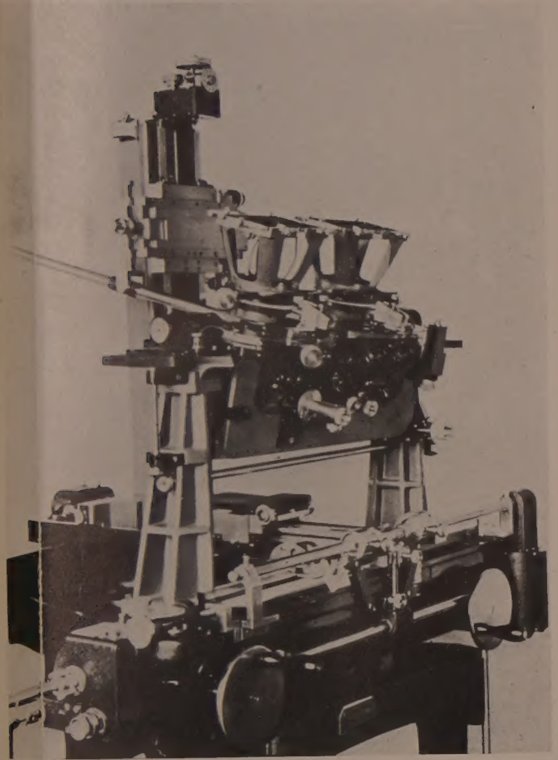
of sufficiently large blocks, and provided that a sufficient ground staff is available to utilize the photographs of such large areas while they are still in date.

I have so far confined my remarks to the preparation of maps, because that is the application of air photography on which I am most qualified to speak. I do not, however, suggest that "surveying" is necessarily confined to "mapping." On the contrary, a great deal of useful information can be obtained by organized examination of air photographs without producing a map, although it will usually be necessary to provide a drawn map from the photographs to record and to exploit the information so gleaned. Air-photographs are, for instance, being used to an increasing extent for geological prospecting, and there are many recorded instances of mineral strikes being made in this way which have been completely missed on the ground, and which could not have been made solely by examination of even the most detailed topographic map. Air photographs may also be used in conjunction with maps prepared from them for a variety of detailed engineering projects, ranging from road or railway alignment to the development of water-power resources. Forest "typing" and stock mapping also benefit from the air photograph, to quote yet another instance. In short, a stereoscopic pair of air-photographs provides the most detailed, yet comprehensive, view of an area of country available to man, and it needs little imagination to realize the value of such a detached view to anyone interested in the natural or artificial surface features of the earth.

NOTE.

The Institution as a body is not responsible either for the statements made, or for the opinions expressed, in the preceding Papers and Lecture.

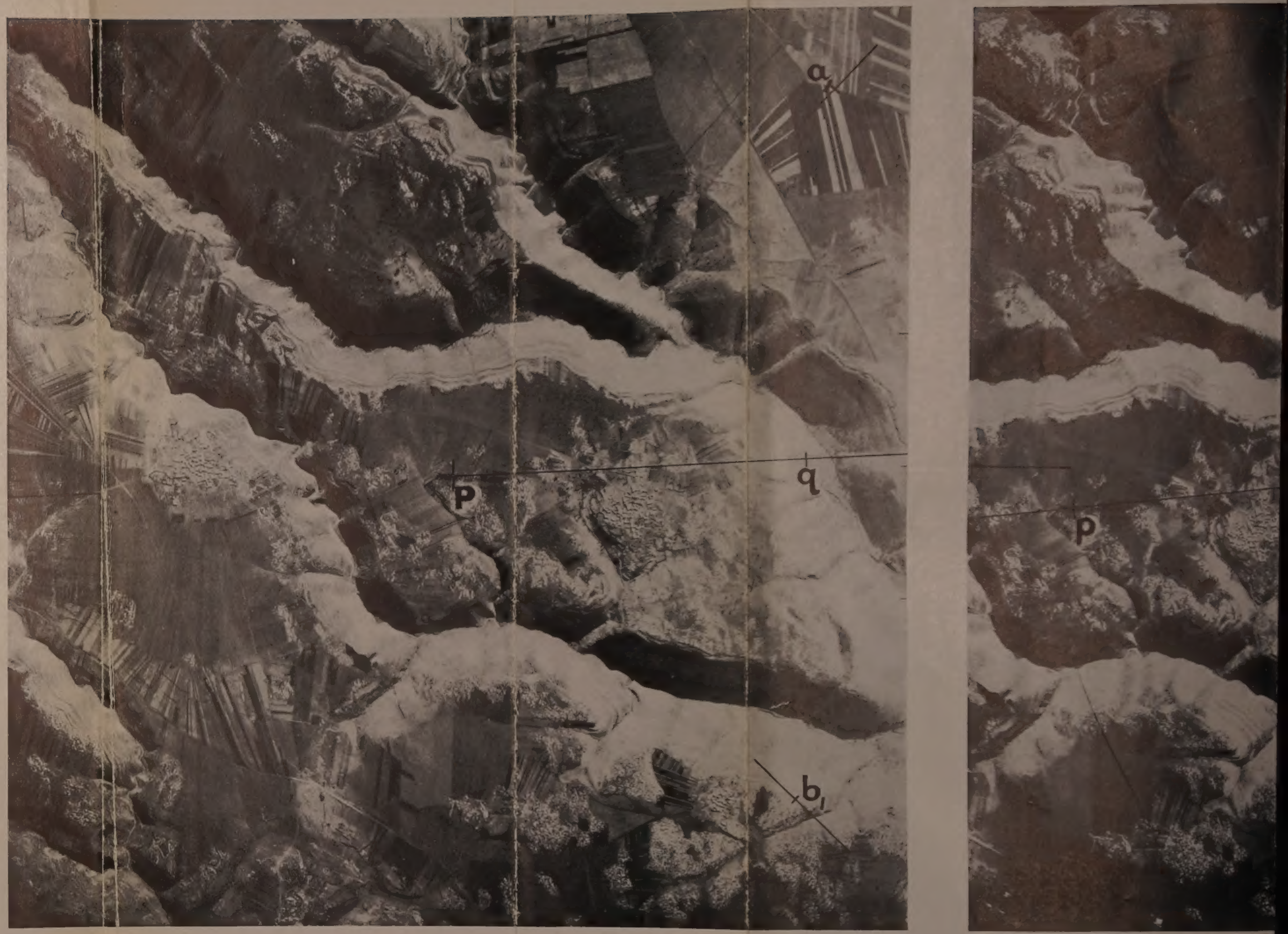
Fig. 2.



WILD AUTOGRAPH.

Fig. 3.

HIGH OBLIQUE PHOTOGRAPH.



SPECIM

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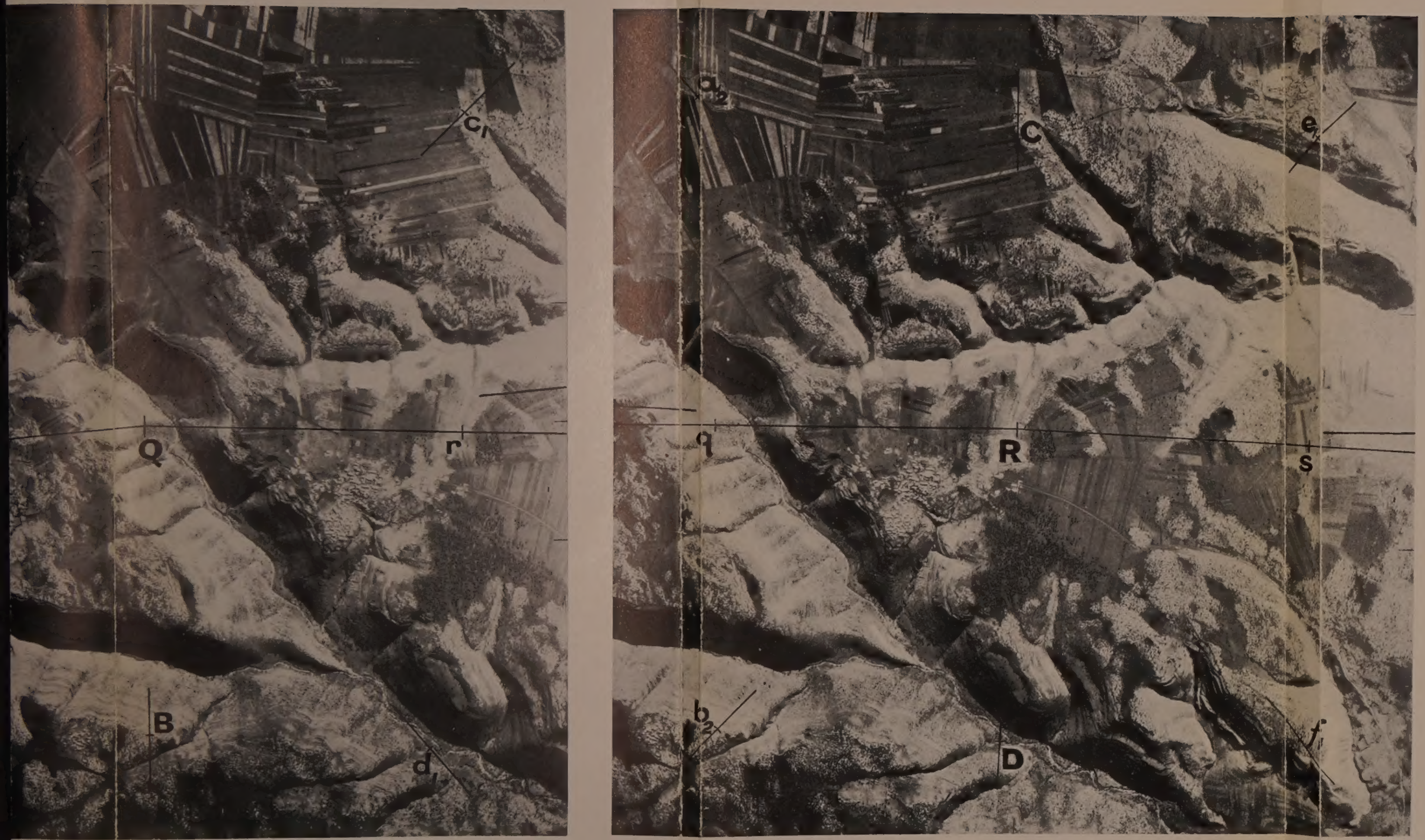
APHS.

Figs. 4.



N SERIES OF VERTICAL PHOTOGRAPHS.

*Royal Air Force Official Photographs—
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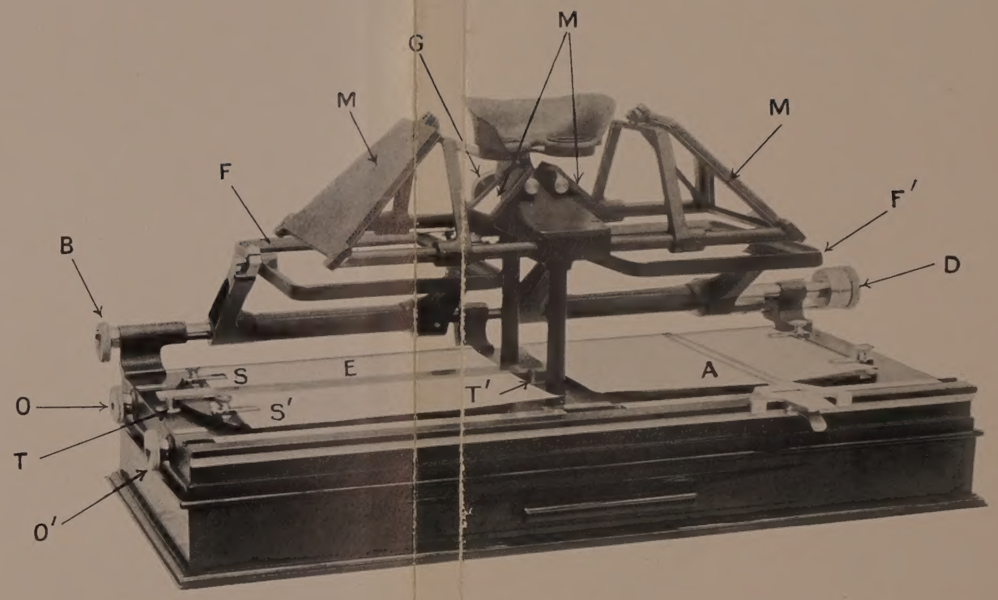
PLATE 1.

SURVEYING FROM AIR PHOTOGRAPHS.

Fig. 5.



TOPOGRAPHICAL STEREOSCOPE
TYPE ZD. 10.

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M. HOTINE.